CHAPTER 7

ALTERNATING CURRENT AND ELECTRICAL MACHINES

7.1 ALTERNATING CURRENT

1. *What is meant by alternating current ? Write an expression for it. Define its amplitude, time period and frequency.*

Alternating current. *An alternating current is that current whose magnitude changes continuously with time and direction reverses periodically.* In contrast to it, a ***direct current*** *is that current which flows with a constant magnitude in the same direction,* as shown in Fig. 7.1.

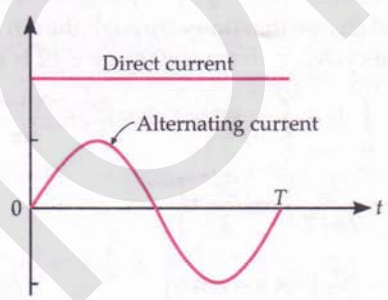


Fig. 7.1 Alternating and direct currents.

We know that when a coil is rotated in a magnetic field, an alternating emf is induced in it, which is given by the relation :

$$\varepsilon = \varepsilon_0 \sin \omega t$$

Suppose this emf is applied to a circuit of resistance *R*. Then by ohm's law, the current in the circuit will be

$$I = \frac{\varepsilon}{R} = \frac{\varepsilon_0}{R} \sin \omega t \quad \text{or} \quad I = I_0 \sin \omega t$$

Thus the current in the circuit varies sinusoidally with time and is called *alternating current.*

Here

I = ***instantaneous value*** of a.c. at any instant t

$I_0 = \frac{\varepsilon_0}{R}$ = ***peak*** or ***maximum value*** of a.c. and is called ***current amplitude.***

Amplitude. *The maximum value attained by an alternating current in either direction is called its* ***amplitude*** *or* ***peak value*** and is denoted by I_0.

Time period. *The time taken by an alternating current to complete one cycle of its variations is called its* time period and is denoted by *T*. This time is equal to the time taken by the coil to complete one rotation in the magnetic field. As angular velocity of the coil is ω and its angular displacement in one complete cycle is 2π, so

$$\text{Time period} = \frac{\text{Angular displacement in a complete cycle}}{\text{Angular velocity}}$$

or $$T = \frac{2\pi}{\omega}$$

Frequency. *The number of cycles completed per second by an alternating current is called its frequency and is denoted by f.* The frequency of an alternating current is same as the frequency of rotation of the coil in the magnetic field. Thus

$$f = \frac{1}{T} = \frac{\omega}{2\pi}$$

So an alternating current can be represented as

$$I = I_0 \sin \omega t = I_0 \sin 2\pi ft = I_0 \sin \frac{2\pi}{T} t$$

Figure 7.1 shows the variation of alternating current with time. It rises from 0 to maximum in one direction, then falls to zero and then rises from 0 to maximum in the opposite direction and again falls to zero, thus completing one full cycle.

The alternating current supplied to our houses has a frequency of 50 cps or 50 Hz.

As the alternating current is positive in one half cycle and equally negative in the other half cycle, so its mean value over a complete cycle is zero. We can prove it mathematically also.

2. *Prove mathematically that the average value of alternating current over one complete cycle is zero.*

Average value of a.c. over one complete cycle. The alternating current at any instant t is given by

$$I = I_0 \sin \omega t$$

Assuming the current remains constant for a small time dt, then the amount of charge that flows through the circuit in small time dt will be

$$dq = I dt = I_0 \sin \omega t \cdot dt$$

The total charge that flows through the circuit in one complete cycle of a.c.,

$$q = \int dq = \int_0^T I_0 \sin \omega t \, dt$$

$$= I_0 \left[\frac{-\cos \omega t}{\omega}\right]_0^T = -\frac{I_0}{2\pi/T}\left[\cos \frac{2\pi}{T} t\right]_0^T$$

$$= -\frac{I_0 T}{2\pi}[\cos 2\pi - \cos 0] = -\frac{I_0 T}{2\pi}[1-1] = 0$$

The average value of a.c. over one complete cycle of a.c.,

$$I_{av} = \frac{q}{T} = \mathbf{0}$$

Thus *the average value of a.c. over a complete cycle of a.c. is zero.*

3. *Ordinary moving coil galvanometer used for d.c. cannot be used to measure an alternating current even if its frequency is low. Explain, why.*

Ordinary moving coil galvanometer cannot be used to measure a.c. Ordinary moving coil galvanometer is based on magnetic effect of current which, in turn, depends on direction of current. So it cannot be used to measure a.c. During one half cycle of a.c., its pointer moves in one direction and during next half cycle, it will move in the opposite direction. Now the average value of a.c. over a complete cycle is zero. Even if we measure an alternating current of low frequency, the pointer, will appear to be stationary at the zero position due to persistence of vision.

We can measure a.c. by using a hot-wire ammeter which is based on heating effect of current and this effect is independent of the direction of current.

To measure a.c., we define the mean value of a.c. over half a cycle or its root mean square value.

7.2 MEAN OR AVERAGE VALUE OF A.C.

4. *Define average value of a.c. over half a cycle. Establish the relationship between the 'average value' and the 'peak value' of an alternating current.*

Average value of a.c. It is defined as that *value of direct current which sends the same charge in a circuit in the same time as is sent by the given alternating current in its half time period.* It is denoted by

$$I_{av} \text{ or } I_m$$

Relation between average value and peak value of a.c. The value of alternating current at any instant t is given by

$$I = I_0 \sin \omega t$$

This current can be assumed to remain constant for a small time dt. Then the amount of charge that flows through the circuit in small time dt is given by

$$dq = I \cdot dt = I_0 \sin \omega t \cdot dt$$

The total charge that flows through the circuit, say in the first half cycle, *i.e.*, from $t = 0$ to $t = T/2$ is given by

$$q = \int_0^{T/2} dq = \int_0^{T/2} I_0 \sin \omega t \, dt = I_0 \left[-\frac{\cos \omega t}{\omega}\right]_0^{T/2}$$

$$= -\frac{I_0}{2\pi/T}\left[\cos \frac{2\pi}{T} t\right]_0^{T/2}$$

$$= -\frac{I_0 T}{2\pi}[\cos \pi - \cos 0] \qquad \left[\because \omega = \frac{2\pi}{T}\right]$$

$$= -\frac{I_0 T}{2\pi}[-1-1] = \frac{I_0 T}{\pi}$$

∴ The average value of a.c. over the first half cycle is

$$I_{av} = \frac{\text{Charge}}{\text{Time}} = \frac{q}{T/2} = \frac{2q}{T} = \frac{2}{T} \cdot \frac{I_0 T}{\pi}$$

or

$$I_{av} = \frac{2}{\pi} I_0 = 0.637 \, I_0$$

Thus the mean or average value of an alternating current is $2/\pi$ or 0.637 times its peak value. The similar relation can be proved for the alternating emf, which is

$$\mathcal{E}_{av} = \frac{2}{\pi}\mathcal{E}_0 = 0.637\,\mathcal{E}_0.$$

7.3 ROOT MEAN SQUARE (RMS) OR VIRTUAL OR EFFECTIVE VALUE OF A.C.

5. *What is meant by rms value or effective value of an alternating current ? Derive a relation between it and its peak value.*

Root mean square or virtual or effective value of a.c. *It is defined as that value of a direct current which produces the same heating effect in a given resistor as is produced by the given alternating current when passed for the same time.* It is denoted by I_{rms}, I_v or by I_{eff}.

Relation between the effective and peak value of a.c. Suppose an alternating current $I = I_0 \sin \omega t$ be passed through a circuit of resistance R. Then the amount of heat produced in small time dt will be

$$dH = I^2 R\, dt$$

If T is the time period of a.c., then heat produced in one complete cycle will be

$$H = \int_0^T I^2 R dt$$

Let I_{eff} be the effective value of a.c. Then heat produced in time T must be

$$H = I_{eff}^2 RT$$

$$\therefore \quad I_{eff}^2 RT = \int_0^T I^2 R\, dt \quad \text{or} \quad I_{eff}^2 = \frac{1}{T}\int_0^T I^2\, dt$$

But $\frac{1}{T}\int_0^T I^2 dt$ is the mean of the squares of the instantaneous values of a.c. over one complete cycle, hence the effective or virtual value of a.c. equals its root mean square value, *i.e.*,

$$I_{eff} = I_{rms} = \sqrt{\frac{1}{T}\int_0^T I^2\, dt}$$

Now $\int_0^T I^2\, dt = \int_0^T I_0^2 \sin^2 \omega t\, dt$

$$= I_0^2 \int_0^T \frac{1 - \cos 2\omega t}{2}\, dt$$

$$= \frac{I_0^2}{2}\left[t - \frac{\sin 2\omega t}{2\omega}\right]_0^T$$

$$= \frac{I_0^2}{2}\left[(T - 0) - \frac{1}{2\omega}\left|\sin \frac{4\pi}{T} t\right|_0^T\right]$$

$$= \frac{I_0^2}{2}\left[T - \frac{1}{2\omega}(\sin 4\pi - \sin 0)\right]$$

$$= \frac{I_0^2}{2}[T - 0] = \frac{I_0^2 T}{2}$$

$$\therefore \quad I_{eff} \text{ or } I_{rms} = \sqrt{\frac{1}{T}\cdot\frac{I_0^2 T}{2}}$$

or $$I_{eff} \text{ or } I_{rms} = \frac{1}{\sqrt{2}} I_0 = 0.707\, I_0$$

Thus the effective or rms value of an a.c. is $\frac{1}{\sqrt{2}}$ times its peak value.

7.4 ROOT MEAN SQUARE VALUE OF AN ALTERNATING EMF

6. *Define the root mean square value of an alternating emf. Derive a relation between it and its peak value.*

Root mean square value of an alternating emf. *It is defined as that value of a steady voltage that produces the same amount of heat in a given resistance as is produced by the given alternating emf when applied to the same resistance for the same time.* It is also called ***virtual*** or ***effective*** value of the alternating emf. It is denoted by $\mathcal{E}_{rms}$ or $\mathcal{E}_{eff}$ or $\mathcal{E}_v$.

Relation between the rms value and the peak value of an alternating emf. Suppose an alternating emf $\mathcal{E}$ applied to a resistance R is given by

$$\mathcal{E} = \mathcal{E}_0 \sin \omega t$$

Heat produced in a small time dt will be

$$dH = \frac{\mathcal{E}^2}{R}\, dt = \frac{\mathcal{E}_0^2}{R}\sin^2 \omega t\, dt$$

Let T be the time period of the alternating emf. Then heat produced in time T will be

$$H = \int dH = \int_0^T \frac{\mathcal{E}_0^2}{R}\sin^2 \omega t\, dt$$

$$= \frac{\mathcal{E}_0^2}{R}\int_0^T \frac{(1 - \cos 2\omega t)}{2}\, dt = \frac{\mathcal{E}_0^2}{2R}\left[t - \frac{\sin 2\omega t}{2\omega}\right]_0^T$$

$$= \frac{\mathcal{E}_0^2}{2R}\left[(T - 0) - \frac{1}{2\omega}\left|\sin\frac{4\pi}{T}t\right|_0^T\right]$$

$$= \frac{\mathcal{E}_0^2}{2R}\left[T - \frac{1}{2\omega}\sin(4\pi - \sin 0)\right]$$

or $$H = \frac{\mathcal{E}_0^2}{2R}[T - 0] = \frac{\mathcal{E}_0^2 T}{2R}$$

If $\mathcal{E}_{rms}$ is the root mean square value of the alternating emf, then the amount of heat produced by it in the same resistance R in the time T will be

$$H = \frac{\mathcal{E}_{rms}^2 T}{R}$$

From the above two equations, we get

$$\frac{\mathcal{E}_{rms}^2 T}{R} = \frac{\mathcal{E}_0^2 T}{2R}$$

or
$$\mathcal{E}_{rms} = \frac{\mathcal{E}_0}{\sqrt{2}} = 0.707\,\mathcal{E}_0$$

For Your Knowledge

- The alternating current and voltages are generally measured and specified in terms of their rms values. When we say that the household supply is 220 V a.c., we mean that its rms value is 220 V. The peak value would be
$$V_0 = \sqrt{2}\,.\,V_{rms} = \sqrt{2} \times 220 = 311\text{ V}.$$
- Both alternating and direct currents are measured in amperes. However, it is not possible to define a.c. ampere in terms of forces between two parallel wires carrying a.c. currents, as the d.c. ampere is defined. This is because the alternating current changes direction with the source frequency and so the net force would add up to zero. To overcome this problem, we define a.c. ampere in terms of Joule heating ($H = I^2Rt$) which is independent of the direction of current. Hence the rms value of alternating current in a circuit is one ampere of the current that produces the same average heating effect as one ampere of direct current would produce under the same conditions.
- Alternating currents and voltages are measured by a.c. ammeter and a.c. voltmeter respectively. As the working of these instruments is based on the heating effect of current, so they are called *hot-wire instruments*.

Examples based on
(i) Mean (ii) Effective (iii) Instantaneous values of Alternating Currents and Voltages

Formulae Used

1. Instantaneous value of a.c., $I = I_0 \sin \omega t$, where I_0 is the peak or maximum value of a.c.
2. Average or mean value of a.c. over half cycle,
$$I_{av} = \frac{2}{\pi} I_0 = 0.637\, I_0$$
3. Effective or rms or virtual value of a.c.,
$$I_{eff} \text{ or } I_{rms} \text{ or } I_v = \frac{1}{\sqrt{2}} I_0 = 0.707\, I_0$$
4. For alternating voltages, we have
$$\mathcal{E} = \mathcal{E}_0 \sin \omega t,\ \mathcal{E}_{av} = 0.637\,\mathcal{E}_0,\ \mathcal{E}_{rms} = \frac{1}{\sqrt{2}}\mathcal{E}_0$$

Units Used

Currents I, I_0 and I_{rms} are in ampere, voltages $\mathcal{E}$, $\mathcal{E}_0$ and $\mathcal{E}_{rms}$ are in volt.

Example 1. *The electric mains in a house are marked 220 V, 50 Hz. Write down the equation for instantaneous voltage.* [CBSE D 95C ; Haryana 02]

Solution. Here $\mathcal{E}_{rms} = 220$ V, $f = 50$ Hz

Instantaneous voltage is given by

$$\mathcal{E} = \mathcal{E}_0 \sin \omega t = \sqrt{2}\,\mathcal{E}_{rms} \sin 2\pi f t$$
$$= 1.414 \times 220 \sin (2 \times 3.14 \times 50\, t)$$
$$= \mathbf{311 \sin 314\, t \text{ volt.}}$$

Example 2. *An electric bulb operates 12 V d.c. If this bulb is connected to an a.c. source and gives normal brightness, what would be the peak value of the source ?*

Solution. For normal brightness of the bulb,

$$\mathcal{E}_{rms} = 12\text{ V}$$
$$\therefore \quad \mathcal{E}_0 = \sqrt{2}\,\mathcal{E}_{rms}$$
$$= 1.414 \times 12 = \mathbf{17\ V.}$$

Example 3. *The peak value of an alternating voltage applied to a 50 Ω resistance is 10 V. Find the rms current. If the voltage frequency is 100 Hz, write the equation for the instantaneous current.*

Solution. Here $R = 50\,\Omega$, $\mathcal{E}_0 = 10$ V, $f = 100$ Hz

$$I_0 = \frac{\mathcal{E}_0}{R} = \frac{10}{50} = \frac{1}{5}\text{ A} = 200\text{ mA}$$
$$I_{rms} = 0.707\ I_0 = 0.707 \times 200 = \mathbf{141.4\ mA}$$

The instantaneous current is given by

$$I = I_0 \sin 2\pi f t = \mathbf{200 \sin 200\,\pi\, t\ mA.}$$

Example 4. *Calculate the rms value of the alternating current shown in Fig. 7.2.* [CBSE D 98]

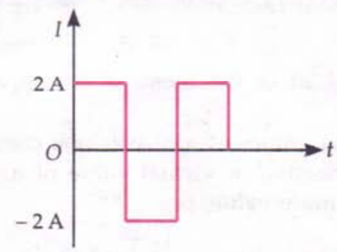


Fig. 7.2

Solution.
$$I_{rms} = \sqrt{\frac{I_1^2 + I_2^2 + I_3^2}{3}}$$
$$= \sqrt{\frac{2^2 + (-2)^2 + 2^2}{3}} = \mathbf{2\ A.}$$

Example 5. *The electric current in a circuit is given by $i = i_0\,(t/\tau)$ for some time. Calculate the rms current for the period $t = 0$ to $t = \tau$.*

Solution. The mean square current for the rms current for the period $t = 0$ to $t = \tau$ is given by

$$\overline{i^2} = \frac{1}{\tau}\int_0^\tau i_0^2\left(\frac{t}{\tau}\right)^2 dt$$

$$= \frac{i_0^2}{\tau^3}\int_0^\tau t^2\, dt = \frac{i_0^2}{\tau^3}\left[\frac{t^3}{3}\right]_0^\tau = \frac{i_0^2}{\tau^3}\cdot\frac{\tau^3}{3} = \frac{i_0^2}{3}$$

$$\therefore i_{rms} = \sqrt{\overline{i^2}} = \sqrt{\frac{i_0^2}{3}} = \frac{i_0}{\sqrt{3}}.$$

Example 6. *If the effective value of current in 50 Hz a.c. circuit is 5.0 A, what is (i) the peak value of current (ii) the mean value of current over half a cycle and (iii) the value of current 1/300 s after it was zero?*

Solution. Here $I_{eff} = 5$ A, $f = 50$ Hz

(*i*) $I_0 = \sqrt{2}\, I_{eff} = \sqrt{2} \times 5 = \mathbf{7.07\ A.}$

(*ii*) $I_m = \frac{2}{\pi} I_0 = 0.637 \times 7.07 = \mathbf{4.5\ A.}$

(*iii*) At $t = 1/300$ s,

$$I = I_0 \sin 2\pi f t = 7.07 \sin\left(2\pi \times 50 \times \frac{1}{300}\right)$$

$$= 7.07 \sin\frac{\pi}{3} = 7.07 \times \frac{\sqrt{3}}{2} = \mathbf{6.12\ A.}$$

Example 7. *The instantaneous value of an alternating voltage in volts is given by the expression* $\mathcal{E}_t = 140 \sin 300\, t$, *where t is in second. What is (i) peak value of the voltage, (ii) its rms value and (iii) frequency of the supply?*

Take $\pi = 3$, $\sqrt{2} = 1.4$.

Solution. Comparing the equation :

$$\mathcal{E}_t = 140 \sin 300\, t$$

with the standard equation : $\mathcal{E} = \mathcal{E}_0 \sin \omega t$, we get

(*i*) Peak voltage, $\mathcal{E}_0 = \mathbf{140\ V.}$

(*ii*) rms value of voltage,

$$\mathcal{E}_{rms} = \frac{\mathcal{E}_0}{\sqrt{2}} = \frac{140}{1.4} = \mathbf{100\ V.}$$

(*iii*) Angular frequency, $\omega = 300$

$$\therefore \text{Frequency,} \quad f = \frac{\omega}{2\pi} = \frac{300}{2 \times 3} = \mathbf{50\ Hz.}$$

Example 8. *A resistance of 40 Ω is connected to an a.c. source of 220 V, 50 Hz. Find (i) the rms current (ii) the maximum instantaneous current in the resistor and (iii) the time taken by the current to change from its maximum value to the rms value.*

Solution. (*i*) $\mathcal{E}_{rms} = 220$ V, $R = 40\ \Omega$

$$\therefore \quad I_{rms} = \frac{\mathcal{E}_{rms}}{R} = \frac{220}{40} = \mathbf{5.5\ A.}$$

(*ii*) Maximum instantaneous current,

$$I_0 = \sqrt{2}\ I_{rms} = 1.414 \times 5.5 = \mathbf{7.8\ A.}$$

(*iii*) Let the alternating current be given by

$$I = I_0 \sin \omega t$$

Let the a.c. take its maximum and rms values at instants t_1 and t_2 respectively. Then

$$I_0 = I_0 \sin \omega t_1,$$

which implies $\omega t_1 = \frac{\pi}{2}$ and $I_{rms} = \frac{I_0}{\sqrt{2}} = I_0 \sin \omega t_2$,

which implies $\omega t_2 = \frac{\pi}{2} + \frac{\pi}{4}$

$$\therefore \quad t_2 - t_1 = \frac{\pi}{4\omega} = \frac{\pi}{4 \times 2\pi f}$$

$$= \frac{\pi}{4 \times 2\pi \times 50} = \frac{1}{400}\ \text{s} = \mathbf{2.5\ ms.}$$

Problems For Practice

1. The instantaneous emf of an a.c. source is given by $\mathcal{E} = 300 \sin 314\, t$. What is the rms value of the emf? **[CBSE D 2000]** (Ans. 212 V)
2. The emf of an a.c. source is given by the expression $\mathcal{E} = 300 \sin 314\, t$. Write the value of peak voltage and frequency of the source. **[CBSE D 93 C]** (Ans. 300 V, 50 Hz)
3. The instantaneous current from an a.c. source is $I = 5 \sin 314\, t$. What is the rms value of current? **[CBSE PMT 2000]** (Ans. 3.54 A)
4. An alternating voltage given by $V = 140 \sin 314t$ is connected across a pure resistor of 50 Ω. Find (*i*) the frequency of the source. (*ii*) the rms current through the resistor. **[CBSE OD 12]** [Ans. (*i*) 50 Hz (*ii*) 1.98 A]
5. An alternating emf of peak value 350 V is applied across an a.c. ammeter of resistance 100 Ω. What is the reading of the ammeter? (Ans. 2.47 A)
6. The effective value of current in a 50 cycle a.c. circuit is 5 A. What is the value of current 1/300 second after it was zero? **[Punjab 94]** (Ans. 6.123 A)
7. The peak value of an alternating current of frequency 50 Hz is 14.14 A. Find its rms value. How much time will the current take in reaching from 0 to maximum value? (Ans. 10 A, 5 ms)
8. A 100 Ω iron is connected to a 220 V, 50 cycles wall plug. What is (*i*) peak potential difference (*ii*) average potential difference and (*iii*) rms current? [Ans. (*i*) 311 V, (*ii*) 198 V (*iii*) 2.2 A]
9. The equation of a.c. in a circuit is $I = 50 \sin 100\,\pi t$. Find (*i*) frequency of a.c., (*ii*) mean value of a.c. over positive half cycle, (*iii*) rms value of current and (*iv*) the value of current 1/300 second after it was zero. (Ans. 50 Hz, 31.8 A, 35.35 A, 43.3 A)

HINTS

1. $\mathcal{E}_{rms} = 0.707,\ \mathcal{E}_0 = 0.707 \times 300 \simeq$ **212 V.**
2. Peak voltage, $\mathcal{E}_0 =$ **300 V.**
$$2\pi f = 314$$
$$f = \frac{314}{2\pi} = \frac{314}{2 \times 3.14} = \mathbf{50\ Hz.}$$
3. $I_0 = 5$ A
$I_{rms} = 0.707\ I_0 = 0.707 \times 5 \simeq$ **3.54 A.**
4. Given $V = 140 \sin 314t = 140 \sin 2\pi ft$
(*i*) $2\pi f = 314$
$$\therefore \quad f = \frac{314}{2\pi} = \frac{314}{2 \times 3.14} = \mathbf{50\ Hz}$$
(*ii*) $I_{\text{rms}} = \dfrac{V_{\text{rms}}}{R} = \dfrac{0.707 V_0}{R} = \dfrac{0.707 \times 140}{50} = \mathbf{1.98\ A}$
5. $I_0 = \dfrac{\mathcal{E}_0}{R} = \dfrac{350}{100} = 3.5$ A
Reading of the ammeter
$$= I_{rms} = 0.707 \times 3.5 = \mathbf{2.47\ A.}$$
6. Current after 1/300 second is given by
$$I = I_0 \sin \omega t = \sqrt{2}\ I_{rms} \sin 2\pi\ f\ t$$
$$= \sqrt{2} \times 5 \sin\left(2\pi \times 50 \times \frac{1}{300}\right)$$
$$= 5\sqrt{2} \sin \frac{\pi}{3} = 5\sqrt{2} \times \frac{\sqrt{3}}{2} = \mathbf{6.123\ A.}$$
7. $I_{rms} = \dfrac{I_0}{\sqrt{2}} = \dfrac{14.14}{1.414} = 10$ A
As $f = 50$ Hz $\therefore\ T = \dfrac{1}{f} = \dfrac{1}{50}$ s
Time taken by the current to reach its maximum value starting from zero $= \dfrac{T}{4} = \dfrac{1}{4 \times 50}$ s $=$ **5 ms.**
8. Here $R = 100\,\Omega,\ \mathcal{E}_{rms} = 220$ V, $f = 50$ Hz
(*i*) $\mathcal{E}_0 = \sqrt{2}\,\mathcal{E}_{rms} = \sqrt{2} \times 220 =$ **311 V.**
(*ii*) $\mathcal{E}_{av} = \dfrac{2}{\pi}\mathcal{E}_0 = 0.637 \times 311 =$ **198 V.**
(*iii*) $I_{rms} = \dfrac{\mathcal{E}_{rms}}{R} = \dfrac{220}{100} =$ **2.2 A.**
9. Comparing $I = 50 \sin 100\,\pi\,t$ with $I = I_0 \sin 2\pi\ f\ t$, we get
(*i*) $2\pi\ f = 100\pi$ or $f = \dfrac{100\,\pi}{2\pi} =$ **50 Hz.**
(*ii*) $I_0 = 50$ A $\therefore\ I_{av}$ or $I_m = 0.637 \times 50 = 31.8$ A.
(*iii*) $I_{rms} = 0.707 \times 50 =$ **35.35 A.**
(*iv*) $I = 50 \sin\left(100\pi \times \dfrac{1}{300}\right) = 50 \sin \dfrac{\pi}{3}$
$$= 50 \times 0.866 = \mathbf{43.3\ A.}$$

7.5 PHASORS AND PHASOR DIAGRAMS

Phasors and phasor diagrams. *A rotating vector that represents a sinusoidally varying quantity is called a* ***phasor***. This vector is imagined to rotate with angular velocity equal to the angular frequency of that quantity. Its length represents the amplitude of the quantity and its projection upon a fixed axis gives the instantaneous value of the quantity. The phase angle between two quantities is shown as the phase angle between their phasors.

The study of a.c. circuits is greatly simplified if we treat alternating currents and voltages as phasors.

A diagram that represents alternating current and voltage of the same frequency as rotating vectors (phasors) alongwith proper phase angle between them is called a ***phasor diagram*** *or* ***Argand diagram***.

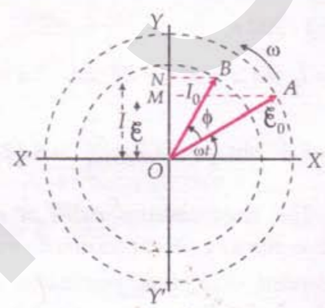


Fig. 7.3 A phasor diagram for an alternating emf and current.

Suppose the alternating emf and current in a circuit are given by
$$\mathcal{E} = \mathcal{E}_0 \sin \omega t \quad \text{and} \quad I = I_0 \sin(\omega t + \phi)$$
where ϕ is the phase angle between $\mathcal{E}$ and I. To represent these quantities as phasors, we draw circles of radii $\mathcal{E}_0$ and I_0 as shown in Fig. 7.3. Let $\angle AOX = \omega t$ and $\angle BOX = \omega t + \phi$.

Then vector $\vec{OA}$ represents phasor $\vec{\mathcal{E}}$ of magnitude $\mathcal{E}_0$ and vector $\vec{OB}$ represents phasor $\vec{I}$ of magnitude I_0, both rotating with the same angular velocity ω in the anticlockwise direction. The projection $OM (= \mathcal{E})$ of $\vec{OA}$ on the vertical axis represents the instantaneous value of the alternating emf. The projection ON $(= I)$ of $\vec{OB}$ on the vertical axis represents the instantaneous value of the alternating current. The angle $\phi = \angle AOB$ represents the phase angle between the phasors $\vec{\mathcal{E}}$ and $\vec{I}$. In the present case, the current leads the emf by phase angle ϕ.

If the current lags behind the emf, we can write
$$I = I_0 \sin(\omega t - \phi)$$

For Your Knowledge

- Though in a phasor diagram, we represent alternating current and voltage as rotating vectors, these quantities are not really vectors themselves. These are scalar quantities. In fact, the amplitudes and phases of the harmonically varying scalars combine mathematically in the same way as do the projections of rotating vectors of corresponding magnitudes and directions. Thus the representation of the harmonically varying quantities as rotating vectors enable us to use the laws of vector addition for adding these quantities.
- In an a.c. circuit, the current may lag behind or lead the voltage, depending on the type of the circuit through which the current flows. This concept is analogous to two cars running at the same speed, with one following the other at a distance. More appropriately, it is like two pendulums of the same frequency which start their motions at different instants of time.

7.6 A.C. CIRCUIT CONTAINING ONLY A RESISTOR

7. *Show that the voltage and current always vary in the same phase in an a.c. circuit containing resistance only. Show the relationship graphically and draw a phasor diagram for it.*

A.C. circuit containing resistance only. As shown in Fig. 7.4, suppose a resistor of resistance R is connected to a source of alternating emf $\mathcal{E}$ given by

$$\mathcal{E} = \mathcal{E}_0 \sin \omega t \quad ...(1)$$

Such a circuit is known as a *purely resistive circuit.*

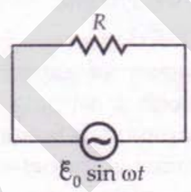


Fig. 7.4 A.C. through a resistor.

If I be the current in the circuit at instant t, then the potential drop across R will be IR. According to Kirchhoff's loop rule,

Instantaneous emf of the source = Instantaneous p.d. across R

or $\mathcal{E}_0 \sin \omega t = IR$

or $$I = \frac{\mathcal{E}_0}{R} \sin \omega t$$

or $$I = I_0 \sin \omega t \quad ...(2)$$

where $I_0 = \frac{\mathcal{E}_0}{R}$ = the maximum or peak value of a.c.

From equations (1) and (2), we note that both $\mathcal{E}$ and I are functions of sin ωt. Hence the emf $\mathcal{E}$ and current I are in *same phase* in a purely resistive circuit. This means that both $\mathcal{E}$ and I attain their zero, minimum and maximum values at the same respective times. This phase relationship is shown graphically in Fig. 7.5(a).

Figure 7.5(b) shows the phasor diagram for a resistive a.c. circuit. Both the phasors $\vec{\mathcal{E}}$ and $\vec{I}$ are in the same direction, making same angle ωt with x-axis. The phase angle between them is zero.

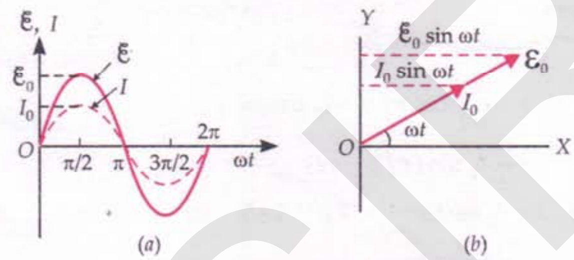


Fig. 7.5 (a) Graph of $\mathcal{E}$ and I versus ωt and (b) Phasor diagram, for a resistive a.c. circuit.

7.7 A.C. CIRCUIT CONTAINING ONLY AN INDUCTOR

8. *A sinusoidal emf is applied to a circuit containing an inductor only. Show that the current lags behind the voltage by π/2 radian. Also derive an expression for the reactance of an inductor, when connected across an a.c. source. Give its units.*

A.C. circuit containing only an inductor. Fig. 7.6 shows an inductor of inductance L connected to a source of alternating emf $\mathcal{E}$ given by

$$\mathcal{E} = \mathcal{E}_0 \sin \omega t \quad ...(1)$$

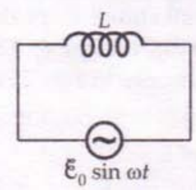


Fig. 7.6 A.C. through an inductor

We assume that the inductor has negligible resistance. Thus the circuit is *purely inductive a.c. circuit.*

As the alternating current flows through the inductor, a back emf $-L\frac{dI}{dt}$ is set up which opposes the applied emf.

∴ Net instantaneous emf $= \mathcal{E} - L\frac{dI}{dt}$

But this emf must be zero because there is no resistance in the circuit.

∴ $$\mathcal{E} - L\frac{dI}{dt} = 0 \quad \text{or} \quad \mathcal{E} = L.\frac{dI}{dt}$$

or $$\mathcal{E}_0 \sin \omega t = L.\frac{dI}{dt}$$

or $$dI = \frac{\mathcal{E}_0}{L} \sin \omega t \, . \, dt$$

Integrating, $$\int dI = \int \frac{\mathcal{E}_0}{L} . \sin \omega t \, . \, dt$$

or $$I = -\frac{\mathcal{E}_0}{\omega L} \cos \omega t + \text{constant}$$

As the applied emf is sinusoidal, we expect the current also to be sinusoidal. Thus the average of current I must be zero over a time period. Now the average of $\cos \omega t$ is zero over a time period, hence the integration constant in the above equation must be zero.

Then

$$I = -\frac{\varepsilon_0}{\omega L} \cos \omega t = -I_0 \cos \omega t$$

$$= -I_0 \sin(\pi/2 - \omega t) \qquad [\because \cos\theta = \sin(\pi/2 - \theta)]$$

or $I = I_0 \sin(\omega t - \pi/2)$ ...(2) $[\because -\sin\theta = \sin(-\theta)]$

where $I_0 = \frac{\varepsilon_0}{\omega L}$ = the peak value of a.c.

Phase relationship between ε and I. On comparing equations (1) and (2), we find that the phase angle of current I is $\pi/2$ rad less than that of emf ε.

Thus in an inductive a.c. circuit, the voltage is ahead of the current in phase by 90° or the current lags behind the voltage in phase by 90°. This means that the voltage ε attains its maximum value (ε_0) a quarter of cycle (time $T/4$) earlier than the current I, or the current attains its peak value (I_0) a quarter of cycle later than the voltage ε. This phase relationship is shown graphically in Fig. 7.7(a).

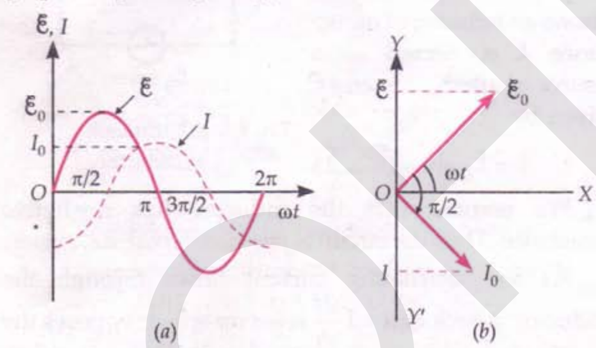


Fig. 7.7 (a) Graph of ε and I versus ωt and (b) phasor diagram, for an inductive a.c. circuit.

Figure 7.7(b) shows the phasor diagram for an inductive a.c. circuit. The phasor $\vec{\varepsilon}$ makes an angle ωt with x-axis in anticlockwise direction. As the current lags behind the emf in phase by $\pi/2$ rad, so the current phasor $\vec{I}$ makes an angle $\pi/2$ rad with the phasor of $\vec{\varepsilon}$ in clockwise direction.

Inductive reactance. Comparing equation $I_0 = \varepsilon_0 / \omega L$ with the ohmic relation $I_0 = \varepsilon_0 / R$, we find that ωL plays the same role here as the resistance R in resistive case. *It is a measure of the effective resistance or opposition offered by the inductor to the flow of a.c. through it.* Such a non-resistive opposition to the flow of current is called ***reactance***. In this case, it is called ***inductive reactance*** and is denoted by X_L.

$$\therefore \qquad X_L = \omega L = 2\pi fL$$

where f is the frequency of a.c. supply. The SI unit of inductive reactance is ohm (Ω).

For a.c., $X_L \propto f$

For d.c., $f = 0$, so $X_L = 0$

Thus an inductor allows d.c. flow through it easily but opposes the flow of a.c. through it. Obviously,

$$I_{rms} = \frac{I_0}{\sqrt{2}} = \frac{\varepsilon_0}{\omega L\sqrt{2}} = \frac{\varepsilon_{rms}}{\omega L} = \frac{\varepsilon_{rms}}{X_L}$$

Variation of X_L with frequency. As $X_L \propto f$, so the graph of X_L versus f is a straight line with a positive slope. As f increases, X_L also increases.

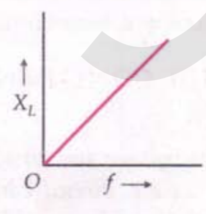


Fig. 7.8 Graph of X_L versus f.

7.8 A.C. CIRCUIT CONTAINING ONLY A CAPACITOR

9. *Explain the effect of introducing a capacitor in d.c. and a.c. circuits.*

Effect of a capacitor in a d.c. circuit. Fig. 7.9 shows a capacitor of capacitance C connected to a battery through a tapping key K. As the circuit is closed, electrons start flowing from the plate A to the positive terminal of the battery and from the negative terminal to the plate B of a capacitor. The plates A and B start acquiring positive and negative charges respectively. The capacitor gets progressively charged until the potential difference across the plates A and B becomes equal to the p.d. across the terminals of the battery. As soon as this happens, the charging of the capacitor stops. Thus, during the capacitor is being charged, an electric current does flow through the rest of the

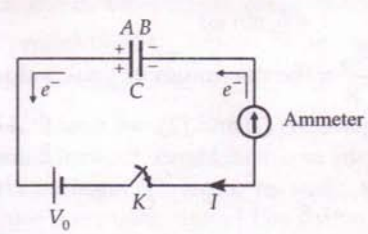


Fig. 7.9 A capacitor in a d.c. circuit.

circuit, as is clear from the momentary deflection in the ammeter. The maximum charge on the capacitor plates will be $q_0 = CV_0$. *Thus a capacitor stops a d.c.*

If a resistance R is also included in series with the capacitor, the process of charging of the capacitor gets slowed down and the capacitor takes longer time to get fully charged. Fig. 7.10 shows the variation of charge q with time t. Clearly, the charge grows exponentially from zero to the maximum value q_0. We may define the ***time constant*** *of the RC-circuit as the time in which the capacitor gets charged to 0.632 times the maximum charge* q_0.

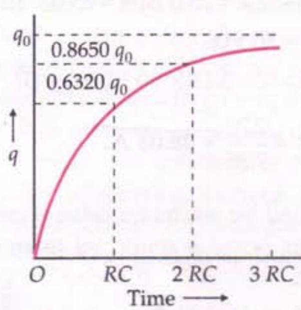


Fig. 7.10 Variation of charge q with time t during the charging of a capacitor

Effect of capacitor in an a.c. circuit. Fig. 7.11 shows a capacitor of capacitor C connected to a source of alternating emf. Due to the alternating voltage of the source, the capacitor gets charged in one direction in the first half cycle, then discharged, and then charged in the opposite direction during the second half cycle and again discharged and so on. As a result, there is a continuous, though alternating, current in the circuit. *Thus a capacitor provides an easy path for a.c.*

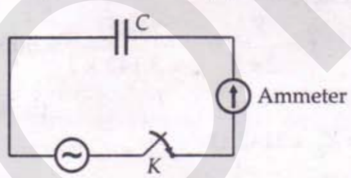


Fig. 7.11 A capacitor in an a.c. circuit.

10. *A sinusoidal emf is applied to a circuit containing a capacitor only. Show that the current leads the voltage by $\pi/2$ radian. Derive the expression for the reactance of a capacitor, when connected across an a.c. source. Give its units.*

A.C. circuit containing only a capacitor. As shown in Fig. 7.12, consider a pure capacitor C connected across a source of alternating emf $\mathcal{E}$ given by

$$\mathcal{E} = \mathcal{E}_0 \sin \omega t \qquad ...(1)$$

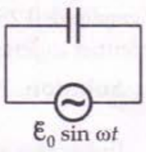


Fig. 7.12

Due to the continuous charging and discharging of the capacitor plates, a continuous but alternating current exists in the circuit.

At any instant,

P.D. across the capacitor plates = Applied emf

i.e., $V = \mathcal{E} = \mathcal{E}_0 \sin \omega t$

But $V = \dfrac{Q}{C}$

or $Q = CV = C\mathcal{E}_0 \sin \omega t$

$\therefore$ Current at any instant is

$$I = \frac{dQ}{dt} = \frac{d}{dt}(C\mathcal{E}_0 \sin \omega t) = \omega C\mathcal{E}_0 \cos \omega t$$

or $$I = I_0 \cos \omega t = I_0 \sin(\omega t + \pi/2) \qquad ...(2)$$

where $I_0 = \omega C\mathcal{E}_0 = \dfrac{\mathcal{E}_0}{1/\omega C}$ = the current amplitude.

Phase relationship between $\mathcal{E}$ and I. *On comparing equations (1) and (2), we find that in a capacitive a.c. circuit, the current leads the voltage or the voltage lags behind the current in phase by $\pi/2$ radian.* The phase relationship between $\mathcal{E}$ and I is shown graphically in Fig. 7.13(a). We see that the current reaches its maximum value earlier than the voltage by one-fourth of a period.

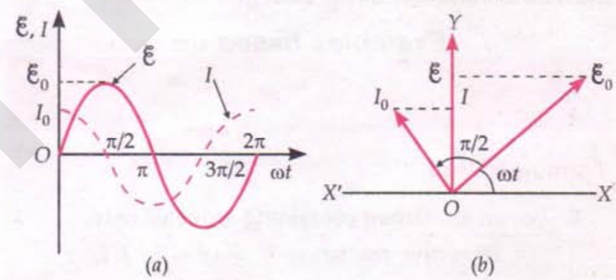


Fig. 7.13 (a) Graph of $\mathcal{E}$ and I versus ωt and (b) Phasor diagram, for a capacitive a.c. circuit.

Figure 7.13(b) shows the phasor diagram for a capacitive a.c. circuit. The phasor $\vec{\mathcal{E}}$ makes an angle ωt with X-axis in anticlockwise direction. As the current leads the emf in phase by $\pi/2$ rad, so the current phasor $\vec{I}$ makes an angle $\pi/2$ rad with phasor $\vec{\mathcal{E}}$ in anticlockwise direction.

Capacitive reactance. Comparing the relation,

$$I_0 = \frac{\mathcal{E}_0}{1/\omega C}$$

with the ohmic relation $I_0 = \dfrac{\mathcal{E}_0}{R}$, we find that the factor $\dfrac{1}{\omega C}$ is the effective resistance or *opposition offered by the capacitor to the flow of a.c. through it*. It is called *capacitive reactance* and is denoted by X_C.

Thus $X_C = \frac{1}{\omega C} = \frac{1}{2\pi fC}$

The SI unit of capacitive reactance is ohm (Ω).

For a.c., $X_C \propto \frac{1}{f}$

For d.c., $f = 0 \quad \therefore \quad X_C = \infty$

Thus a capacitor allows a.c. to flow through it easily but offers infinite resistance to the flow of d.c., *i.e, a capacitor blocks d.c.* Obviously,

$$I_{rms} = \frac{I_0}{\sqrt{2}} = \frac{\varepsilon_0}{1/\omega C \cdot \sqrt{2}} = \frac{\varepsilon_{rms}}{1/\omega C} = \frac{\varepsilon_{rms}}{X_C}$$

Variation of capacitive reactance with frequency. Capacitive reactance,

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi fC}$$

i.e., $X_C \propto \frac{1}{f}$

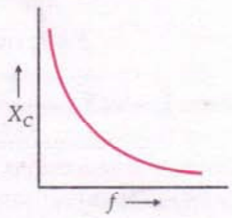


Fig. 7.14 Graph of X_C vs. f

Thus the capacitive reactance varies inversely with the frequency. As f increases, X_C decreases. Fig. 7.14 shows the variation of X_C with f.

Examples based on

(i) Inductive reactance
(ii) Capacitive reactance

Formulae Used

1. For an a.c. circuit containing inductor only,
   (*i*) Inductive reactance, $X_L = \omega L = 2\pi f L$
   (*ii*) Current amplitude, $I_0 = \frac{\varepsilon_0}{X_L} = \frac{\varepsilon_0}{\omega L}$
   (*iii*) Effective current, $I_{rms} = \frac{\varepsilon_{rms}}{X_L} = \frac{\varepsilon_{rms}}{\omega L} = \frac{\varepsilon_0}{\sqrt{2} \cdot \omega L}$
2. For an a.c. circuit containing capacitor only,
   (*i*) Capacitive reactance, $X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$
   (*ii*) Current amplitude, $I_0 = \frac{\varepsilon_0}{X_C} = \frac{\varepsilon_0}{1/\omega C}$
   (*iii*) Effective current,
   $$I_{rms} = \frac{\varepsilon_{rms}}{X_C} = \frac{\varepsilon_{rms}}{1/\omega C} = \frac{\varepsilon_0}{\sqrt{2} \cdot 1/\omega C}$$

Units Used

Inductance L is in henry, capacitance C in farad, reactances X_L and X_C in ohm, currents I_0 and I_{rms} in ampere and voltages ε_0 and ε_{rms} in volt.

Example 9. *A 100 Hz a.c. is flowing in a 14 mH coil. Find its reactance.* [Haryana 98]

Solution. Here $f = 100$ Hz, $L = 14$ mH $= 14 \times 10^{-3}$ H

Reactance, $X_L = 2\pi f L$

$$= 2 \times \frac{22}{7} \times 100 \times 14 \times 10^{-3} = \mathbf{8.8\ \Omega}.$$

Example 10. *A pure inductor of 25.0 mH is connected to a source of 220 V. Find the inductive reactance and rms current in the circuit if the frequency of the source is 50 Hz.* [NCERT]

Solution. Here, $L = 25.0$ mH $= 25.0 \times 10^{-3}$ H, $\varepsilon_{rms} = 220$ V, $f = 50$ Hz

$$X_L = 2\pi fL = 2 \times 3.14 \times 50 \times 25.0 \times 10^{-3} = \mathbf{7.85\ \Omega}$$

$$I_{rms} = \frac{\varepsilon_{rms}}{X_L} = \frac{220}{7.85} = \mathbf{28.03\ A.}$$

Example 11. *Find the maximum value of current when an inductance of one henry is connected to an a.c. source of 200 volts, 50 Hz.* [CBSE OD 95 C ; Punjab 2000]

Solution. Here $L = 1$ H, $\varepsilon_{eff} = 200$ V, $f = 50$ Hz

Maximum current,

$$I_0 = \frac{\varepsilon_0}{X_L} = \frac{\sqrt{2} \times \varepsilon_{eff}}{2\pi f L} = \frac{\sqrt{2} \times 200}{2 \times 3.14 \times 50 \times 1} = \mathbf{0.9\ A.}$$

Example 12. *A coil has an inductance of 1 H. (i) At what frequency will it have a reactance of 3142 Ω ? (ii) What should be the capacity of a capacitor which has the same reactance at that frequency ?* [CBSE D 95 C]

Solution. (*i*) Here $L = 1$ H, $X_L = 3142\ \Omega$

$\therefore$ Frequency,

$$f = \frac{X_L}{2\pi L} = \frac{3142}{2 \times 3.142 \times 1} = \mathbf{500\ Hz}$$

$[\because X_L = 2\pi f L]$

(*ii*) $X_C = X_L = 3142\ \Omega$

But $X_C = \frac{1}{2\pi f C}$

$$\therefore \quad C = \frac{1}{2\pi f X_C} = \frac{1}{2 \times 3.142 \times 500 \times 3142}$$

$$= 0.11 \times 10^{-6}\text{F} = \mathbf{0.11\ \mu F.}$$

Example 13. *An a.c. circuit consists of only an inductor of inductance 2 H. If the current is represented by a sine wave of amplitude 0.25 A and frequency 60 Hz, calculate the effective potential difference (V_{eff}) across the inductor.* ($\pi = 3.14$)

Solution. Here $L = 2$ H, $I_0 = 0.25$ A, $f = 60$ Hz

Inductive reactance, $X_L = \frac{V_{eff}}{I_{eff}}$

$$\therefore \quad V_{eff} = X_L \cdot I_{eff} = 2\pi fL \cdot \frac{I_0}{\sqrt{2}}$$

$$= 2 \times 3.14 \times 60 \times 2 \times \frac{0.25}{1.414} \text{ V} = \mathbf{133.2 \ V.}$$

Example 14. *Alternating emf, $\mathcal{E} = 220 \sin 100 \pi t$ is applied to a circuit containing an inductance of $1/\pi$ H. Write an equation for instantaneous current through the circuit. What will be the reading of an a.c. ammeter if connected in the circuit ?*

Solution. Alternating emf, $\mathcal{E} = 220 \sin 100 \pi t$

Comparing with $\mathcal{E} = \mathcal{E}_0 \sin 2\pi ft$, we get

$$\mathcal{E}_0 = 220 \text{ V}, \ f = 50 \text{ Hz}$$

Current amplitude,

$$I_0 = \frac{\mathcal{E}_0}{\omega L} = \frac{\mathcal{E}_0}{2\pi fL} = \frac{220}{2\pi \times 50 \times \frac{1}{\pi}} = 2.2 \text{ A}$$

Since the current in an inductive circuit lags behind the emf in phase by $\frac{\pi}{2}$ radian, therefore, instantaneous current through the circuit is

$$I = I_0 \sin(100 \pi t - \pi/2)$$

$$= 2.2 \sin(100 \pi t - \pi/2)$$

The a.c. ammeter will read the *rms* value of current,

$$I_{rms} = \frac{I_0}{\sqrt{2}} = \frac{2.2}{\sqrt{2}} = \mathbf{1.556 \ A.}$$

Example 15. *An inductor of inductance 200 mH is connected to an a.c. source of peak emf 210 V and frequency 50 Hz. Calculate the peak current. What is the instantaneous voltage of the source when the current is at its peak value ?*

Solution. Here $L = 200 \text{ mH} = 0.2 \text{ H}, \ \mathcal{E}_0 = 210 \text{ V}, \ f = 50 \text{ Hz}$

Peak current,

$$I_0 = \frac{\mathcal{E}_0}{X_L} = \frac{\mathcal{E}_0}{2\pi f L}$$

$$= \frac{210}{2 \times 3.14 \times 50 \times 0.2} = \mathbf{3.3 \ A}$$

As in an inductive a.c. circuit, current lags behind the emf by $\pi/2$, so the voltage is zero when the current is at its peak value.

Example 16. *A 1.50 μF capacitor is connected to a 220 V, 50 Hz source. Find the capacitive reactance and the current (rms and peak) in the circuit. If the frequency is doubled, what happens to the capacitive reactance and the current ?* [NCERT ; CBSE OD 09]

Solution. Here $C = 1.50 \ \mu\text{F} = 1.50 \times 10^{-6}$ F, $\mathcal{E}_{rms} = 220$ V, $f = 50$ Hz

Capacitive reactance,

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2 \times 3.14 \times 50 \times 1.50 \times 10^{-6}} = \mathbf{212 \ \Omega}$$

$$I_{rms} = \frac{\mathcal{E}_{rms}}{X_C} = \frac{220}{212} = \mathbf{1.04 \ A}$$

Peak current, $I_0 = \sqrt{2} \ I_{rms} = 1.414 \times 1.04 = \mathbf{1.47 \ A}$

The current in the circuit oscillates between + 1.47 A and − 1.47 A and is ahead of emf by 90°.

Now $\quad X_C \propto \frac{1}{f}$

If frequency is doubled, the capacitive reactance is halved and consequently, the current is doubled.

Example 17. *A capacitor of 1 μF is connected to an a.c. source of emf $\mathcal{E} = 250 \sin 100 \pi t$. Write an equation for instantaneous current through the circuit and give reading of a.c. ammeter connected in the circuit.*

Solution. Here $C = 1 \ \mu\text{F} = 10^{-6}$ F, $\mathcal{E}_0 = 250$ V, $\omega = 100 \pi \text{ rad s}^{-1}$.

The instantaneous current through the circuit,

$$I = I_0 \sin\left(\omega t + \frac{\pi}{2}\right) = \omega C \mathcal{E}_0 \sin\left(\omega t + \frac{\pi}{2}\right)$$

$$= 2 \times 3.14 \times 50 \times 10^{-6} \times 250 \sin\left(100 \pi t + \frac{\pi}{2}\right)$$

$$= 0.0786 \sin\left(100 \pi t + \frac{\pi}{2}\right)$$

Reading of the a.c. ammeter is

$$I_{rms} = 0.707 \ I_0 = 0.707 \times 0.0786 \simeq \mathbf{0.06 \ A.}$$

Problems For Practice

1. What is the inductive reactance of a coil if current through it is 800 mA and the voltage across it is 40 V ? (Ans. 50 Ω)
2. Find the value of current through an inductance of 2.0 H and negligible resistance, when connected to an a.c. source of 150 V and 50 Hz. **[Punjab 92]** (Ans. 0.239 A)
3. An inductance of negligible resistance, whose reactance is 22 Ω at 200 Hz is connected to a 220 V, 50 Hz power line. What is the value of inductance and reactance ? (Ans. 0.0175 H, 5.5 Ω)
4. A coil of self-inductance has inductive reactance of 88 Ω. Calculate the self-inductance of the coil if the frequency is 50 Hz. (Ans. 0.28 H)
5. Find the maximum current through an inductance of 2 H connected to an a.c. source of 150 V, 50 Hz. **[Punjab 97]** (Ans. 0.337 A)
6. Calculate the frequency at which the inductive reactance of 0.7 H inductor is 220 Ω. (Ans. 50 Hz)

7. What is the capacitive reactance of a 5 μF capacitor when it is a part of a circuit whose frequency is (*i*) 50 Hz (*ii*) 10^6 Hz ? **[Haryana 95]** (Ans. 636.6 Ω, 3.18×10^{-2} Ω)

8. A capacitor has a capacitance of $1/\pi$ μF. Find its reactance for a frequency of (*i*) 50 Hz and (*ii*) 10^6 Hz. (Ans. 10 Ω, 0.5 Ω)

9. A 1.5 μF capacitor has a capacitive reactance of 12 Ω. What is the frequency of the source ? If the frequency of the source is doubled, what will be the capacitive reactance ? (Ans. 8846 Hz, 6 Ω)

10. A capacitor of capacitance 10 μF is connected to an oscillator giving an output voltage, $\mathcal{E} = 10 \sin \omega t$ volt. If $\omega = 10$ rad s^{-1}, find the peak current in the circuit. (Ans. 1.0 mA)

11. A capacitor has a reactance of 100 Ω at 50 Hz. What will be its reactance at 125 Hz ? (Ans. 40 Ω)

HINTS

1. $X_L = \frac{\mathcal{E}_{eff}}{I_{eff}} = \frac{40}{800 \times 10^{-3}} = \mathbf{50\,\Omega.}$

2. Here $L = 2.0$ H, $\mathcal{E}_{eff} = 150$ V, $f = 50$ Hz

Reactance, $X_L = 2\pi f L = 2 \times 3.14 \times 50 \times 2 = 628\,\Omega$

Current, $I_{eff} = \frac{\mathcal{E}_{eff}}{X_L} = \frac{150}{628} = \mathbf{0.239 A.}$

3. At $f = 200$ Hz, $X_L = 22\,\Omega$

$\therefore \quad L = \frac{X_L}{2\pi f} = \frac{22 \times 7}{2 \times 22 \times 200} = \frac{7}{400} = \mathbf{0.0175\ H.}$

At $f = 50$ Hz, $X_L = 2\pi f L = \frac{2 \times 22 \times 50 \times 7}{7 \times 400} = \mathbf{5.5\,\Omega.}$

4. $L = \frac{X_L}{2\pi f} = \frac{88 \times 7}{2 \times 22 \times 50} = \mathbf{0.28\ H.}$

5. $X_L = 2\pi f L = 2 \times \frac{22}{7} \times 50 \times 2 = \frac{4400}{7}\,\Omega$

$I_0 = \frac{\mathcal{E}_0}{X_L} = \frac{\sqrt{2}\,\mathcal{E}_{rms}}{X_L} = \frac{1.414 \times 150 \times 7}{4400} = \mathbf{0.337\ A.}$

6. $f = \frac{X_L}{2\pi L} = \frac{220 \times 7}{2 \times 22 \times 0.7} = \mathbf{50\ Hz.}$

7. Capacitive reactance, $X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$

(*i*) When $f = 50$ Hz,

$$X_C = \frac{1}{2 \times 3.14 \times 50 \times 5 \times 10^{-6}}\,\Omega = \mathbf{636.6\,\Omega}$$

(*ii*) When $f = 10^6$ Hz,

$$X_C = \frac{1}{2 \times 3.14 \times 10^6 \times 5 \times 10^{-6}}\,\Omega$$

$$= \mathbf{3.18 \times 10^{-2}\,\Omega.}$$

8. Proceed as in Problem 7 above.

9. Here $C = 1.5\,\mu F = 1.5 \times 10^{-6}$ F, $X_C = \frac{1}{2\pi f C} = 12\,\Omega$

$$\therefore \quad f = \frac{1}{2\pi X_C C}$$

$$= \frac{1}{2 \times 3.14 \times 10^{-6} \times 12} = \mathbf{8846\ Hz}$$

When frequency is doubled, capacitive reactance becomes,

$$X'_C = \frac{1}{2\pi \times 2f \times C} = \frac{X_C}{2} = \mathbf{6\,\Omega.}$$

10. $I_0 = \frac{\mathcal{E}_0}{1/\omega C} = \omega C \mathcal{E}_0$

$= 10 \times 10 \times 10^{-6} \times 10 = 10^{-3}$ A

$= \mathbf{1.0\ mA.}$

11. As $X_C \propto \frac{1}{f}$

$$\therefore \quad \frac{X'_C}{X_C} = \frac{f}{f'}$$

or $$X'_C = \frac{f}{f'} . X_C = \frac{50 \times 100}{125} = \mathbf{40\,\Omega.}$$

7.9 A.C. CIRCUIT WITH RESISTANCE AND INDUCTANCE IN SERIES

11. *An alternating emf is applied to a series combination of a resistor and a capacitor. Investigate the phase relationship between current and emf. Find the impedance of the circuit.*

A.C. circuit containing *L* and *R* in series. As shown in Fig. 7.15, consider a resistor *R* and inductance *L* connected in series to a source of alternating emf $\mathcal{E}$ given by $\mathcal{E} = \mathcal{E}_0 \sin \omega t$.

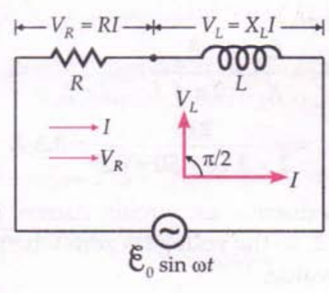


Fig. 7.15 A series *LR*-circuit.

Let *I* be the current through the series circuit at any instant. Then

1. Voltage $\vec{V}_R = R\vec{I}$ across the resistance *R* will be in phase with current $\vec{I}$. So phasors $\vec{V}_R$ and $\vec{I}$ are in same direction, as shown in Fig. 7.16. The amplitude of $\vec{V_R}$ is

$$V_0^R = I_0 R$$

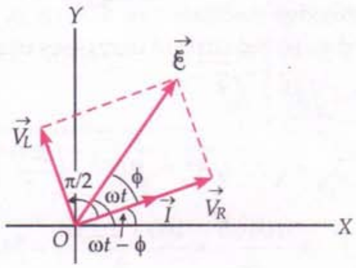


Fig. 7.16 Phasor diagram for series *LR*-circuit.

2. Voltage $\vec{V_L} = X_L \vec{I}$ across the inductance L is *ahead* of current $\vec{I}$ in phase by $\pi/2$ rad. So phasor $\vec{V_L}$ lies $\pi/2$ rad anticlockwise w.r.t. the phasor $\vec{I}$. Its amplitude is

$$V_0^L = I_0 X_L$$

where X_L is the inductive reactance.

By parallelogram law of vector addition,

$$\vec{V_R} + \vec{V_L} = \vec{\mathcal{E}}$$

Using Pythagorean theorem, we get

$$\mathcal{E}_0^2 = (V_0^R)^2 + (V_0^L)^2 = (I_0 R)^2 + (I_0 X_L)^2$$

$$= I_0^2 (R^2 + X_L^2)$$

or

$$I_0 = \frac{\mathcal{E}_0}{\sqrt{R^2 + X_L^2}}$$

Clearly, $\sqrt{R^2 + X_L^2}$ is the effective resistance of the series *LR*-circuit which opposes or impedes the flow of a.c. through it. It is called its ***impedance*** and is denoted by Z. Thus

$$Z = \sqrt{R^2 + X_L^2} = \sqrt{R^2 + \omega^2 L^2} \qquad [\because X_L = \omega L]$$

The phase angle ϕ between the resultant voltage and current is given by

$$\tan\phi = \frac{V_0^L}{V_0^R} = \frac{I_0 X_L}{I_0 R} = \frac{X_L}{R} = \frac{\omega L}{R}$$

It is obvious from the phasor diagram that the current lags behind the emf by phase angle ϕ, so the instantaneous value of current is given by

$$I = I_0 \sin(\omega t - \phi)$$

Examples based on Series LR-circuit

Formulae Used

1. Impedance, $Z = \dfrac{\mathcal{E}_{rms}}{I_{rms}} = \sqrt{R^2 + X_L^2} = \sqrt{R^2 + \omega^2 L^2}$
2. Current, $I_{rms} = \dfrac{\mathcal{E}_{rms}}{Z}$
3. Phase angle ϕ is given by

$$\tan\phi = \frac{X_L}{R} = \frac{\omega L}{R} \quad \text{or} \quad \cos\phi = \frac{R}{Z}.$$

4. Instantaneous current, $I = I_0 \sin(\omega t - \phi)$

Units Used

R, X_L and Z are all in ohm, inductance L in henry and angular frequency ω in rad s^{-1}.

Example 18. *When an inductor L and a resistor R in series are connected across a 12 V, 50 Hz supply, a current of 0.5 A flows in the circuit. The current differs in phase from applied voltage by π/3 radian. Calculate the value of R.* [CBSE OD 06]

Solution. Here, $\mathcal{E}_{rms} = 12$ V, $f = 50$ Hz, $I_{rms} = 0.5$ A, $\phi = \pi/3$ rad

Impedance, $Z = \dfrac{\mathcal{E}_{rms}}{I_{rms}} = \dfrac{12}{0.5} = 24\,\Omega$

As $\cos\phi = \dfrac{R}{Z}$

$$\therefore \quad R = Z\cos\phi = 24\cos\frac{\pi}{3} = 24 \times \frac{1}{2} = \mathbf{12\,\Omega}.$$

Example 19. *A bulb of resistance 10 Ω, connected to an inductor of inductance L, is in series with an a.c. source marked 100 V, 50 Hz. If the phase angle between the voltage and current is π/4 radian, calculate the value of L.* [CBSE OD 01]

Solution. Here $R = 10\,\Omega$, $f = 50$ Hz, $\phi = \dfrac{\pi}{4}$ rad

As $\tan\phi = \dfrac{X_L}{R} = \dfrac{2\pi f L}{R}$

$$\therefore \quad L = \frac{R\tan\phi}{2\pi f} = \frac{10 \times \tan\pi/4}{2 \times 3.142 \times 50} = \mathbf{0.0318\ H.}$$

Example 20. *A coil of resistance 300 Ω and inductance 1.0 H is connected across an alternating voltage of frequency 300/2 π Hz. Calculate the phase difference between the voltage and current in the circuit.*

Solution. Here $R = 300\,\Omega$, $L = 1.0$ H, $f = \dfrac{300}{2\pi}$ Hz

$$\tan\phi = \frac{\omega L}{R} = \frac{2\pi f L}{R} = \frac{2\pi \times 300 \times 1.0}{2\pi \times 300} = 1$$

$\therefore$ Phase difference, $\phi = \mathbf{45°}$.

Example 21. *A coil 'when connected across a 10 V d.c. supply draws a current of 2 A. When it is connected across a 10 V – 50 Hz a.c. supply, the same coil draws a current of 1 A. Explain why it draws lesser current in the second case. Hence determine the self inductance of the coil. (Take $\pi = 3$).* [CBSE Sample Paper 03]

Solution. The coil draws lesser current in the second case because of the reactance offered by the inductor.

In case of d.c., $V = 10$ V, $I = 2$ A

$$\therefore \quad R = \frac{V}{I} = \frac{10}{2} = 5\,\Omega$$

In case of a.c., $\varepsilon_{eff} = 10$ V, $I_{eff} = 1$ A

$$\therefore \quad Z = \frac{\varepsilon_{eff}}{I_{eff}} = \frac{10}{1} = 10\,\Omega$$

Inductive reactance,

$$X_L = \sqrt{Z^2 - R^2} = \sqrt{10^2 - 5^2} = 5\sqrt{3}\,\Omega$$

or $$2\pi f L = 5\sqrt{3}$$

$$\therefore \quad L = \frac{5\sqrt{3}}{2\pi f} = \frac{5\sqrt{3}}{2 \times 3 \times 50} = \mathbf{0.0288\ H.}$$

Example 22. *An 80 V, 800 W heater is to be operated on a 100 V, 50 Hz supply. Calculate the inductance of the choke required.* [CBSE D 91 ; Haryana 02]

Solution. As $P = VI$

$$\therefore \quad I = \frac{P}{V} = \frac{800}{80} = 10 \text{ A} \quad \text{and} \quad R = \frac{V}{I} = \frac{80}{10} = 8\,\Omega$$

As the choke is connected in series with the heater, the current should remain same for the impedance adjusted.

$$\therefore \quad I_{eff} = \frac{V_{eff}}{\sqrt{R^2 + \omega^2 L^2}} = \frac{V_{eff}}{\sqrt{R^2 + 4\pi^2 f^2 L^2}}$$

or $$10 = \frac{100}{\sqrt{8^2 + 4\pi^2 \times 50^2 \times L^2}}$$

or $$64 + 10000\,\pi^2 L^2 = 100$$

or $$L^2 = \frac{36}{10000\,\pi^2} \quad \text{or} \quad L = \frac{6}{100\,\pi} = \mathbf{0.019\ H.}$$

Example 23. *A student connects a long air core coil of manganin wire to a 100 V d.c. source and records a current of 1.5 A. When the same coil is connected across 100 V, 50 Hz a.c. source the current reduces to 1.0 A.*

(i) Give reason for this observation.

(ii) Calculate the value of the reactance of the coil. [CBSE D 94]

Solution. (*i*) For d.c. circuit, resistance

$$R = \frac{V}{I} = \frac{100}{1.5} = \frac{200}{3} = 66.7\,\Omega$$

For a.c. circuit, impedance

$$Z = \frac{V_{eff}}{I_{eff}} = \frac{100}{1} = 100\,\Omega$$

As the effective resistance of the coil is greater for a.c. than for d.c., so the current decreases in a.c. circuit.

(*ii*) As $Z = \sqrt{R^2 + X_L^2}$

$$\therefore \quad X_L = \sqrt{Z^2 - R^2} = \sqrt{100^2 - \left(\frac{200}{3}\right)^2}$$

$$= \frac{100\sqrt{5}}{3} = \frac{100 \times 2.2361}{3} = \mathbf{74.53\,\Omega.}$$

Example 24. *When 200 volts d.c. are applied across a coil, a current of 2 ampere flows through it. When 200 volts a.c. of 50 cps are applied to the same coil, only 1.0 ampere flows. Calculate the resistance, impedance and inductance of the coil.* [CBSE F 95]

Solution. (*i*) For d.c. circuit, $V = 200$ V, $I = 2$ A

$$\therefore \text{ Resistance, } R = \frac{V}{I} = \frac{200}{2} = \mathbf{100\,\Omega}$$

(*ii*) For a.c. circuit, $\varepsilon_{eff} = 200$ V, $I_{eff} = 1.0$ A, $f = 50$ Hz.

$$\therefore \text{ Impedance, } Z = \frac{\varepsilon_{eff}}{I_{eff}} = \frac{200}{1.0} = \mathbf{200\,\Omega}.$$

(*iii*) Let L be the inductance of the coil. Then

$$\omega^2 L^2 = Z^2 - R^2 = 200^2 - 100^2 = 30{,}000$$

$$[\because Z = \sqrt{R^2 + \omega^2 L^2}]$$

or $$\omega L = 100\sqrt{3}\,\Omega$$

$$\therefore \quad L = \frac{100\sqrt{3}}{\omega} = \frac{100\sqrt{3}}{2\pi f}$$

$$= \frac{100\sqrt{3}}{2 \times 3.14 \times 50} = \mathbf{0.55\ H.}$$

Example 25. *A 60 – 10 W electric lamp is to be run on 100 V – 60 Hz mains. (i) Calculate the inductance of the choke required. (ii) If a resistor is to be used in place of choke coil to achieve the same result, calculate its value.* [CBSE D 97]

Solution. Here $\varepsilon_{eff} = 60$ V, $P = 10$ W

Resistance of the lamp,

$$R = \frac{\varepsilon_{eff}^2}{P} = \frac{60 \times 60}{10} = 360\,\Omega$$

Current through the lamp,

$$I_{eff} = \frac{P}{\varepsilon_{eff}} = \frac{10}{60} = \frac{1}{6} \text{ A}$$

(*i*) $\mathcal{E}'_{eff} = 100$ V, $f = 60$ Hz

Required impedance,

$$Z = \frac{\mathcal{E}'_{eff}}{I_{eff}} = \frac{100}{1/6} = 600\,\Omega$$

Reactance of required choke $= \sqrt{Z^2 - R^2}$.

or $$X_L = \sqrt{600^2 - 360^2} = 480\,\Omega$$

Inductance of required choke,

$$L = \frac{X_L}{2\pi f} = \frac{480}{2 \times 31.4 \times 60} = \mathbf{1.273\ H.}$$

(*ii*) Value of resistance required in place of choke

$$= 600 - 360 = \mathbf{240\,\Omega}.$$

Example 26. *A 12 Ω resistance and an inductance of 0.05 / π H are connected in series. Across the ends of the circuit is connected a 130 V a.c. supply of 50 Hz. Calculate (i) the current in the circuit and (ii) phase difference between the current and voltage.* [Haryana 03]

Solution. Here $R = 12\,\Omega$, $L = \frac{0.05}{\pi}$ H, $\mathcal{E}_{rms} = 130$ V, $f = 50$ Hz.

Impedance of the *LR*-circuit,

$$Z = \sqrt{R^2 + \omega^2 L^2} = \sqrt{R^2 + 4\pi^2 f^2 L^2}$$

$$= \sqrt{12^2 + 4\pi^2 \times 2500 \times \frac{25 \times 10^{-4}}{\pi^2}}$$

$$= \sqrt{144 + 25} = \sqrt{169} = 13\,\Omega$$

(*i*) Current in the circuit,

$$I_{rms} = \frac{\mathcal{E}_{rms}}{Z} = \frac{130}{13} = \mathbf{10\ A.}$$

(*ii*) Phase difference ϕ is given by

$$\tan\phi = \frac{\omega L}{R} = \frac{2\pi f L}{R} = \frac{2\pi \times 50 \times 0.05}{12 \times \pi} = 0.4167$$

$$\therefore \quad \phi = \tan^{-1}(0.4167) = \mathbf{22.6°}.$$

Here the voltage leads the current by a phase angle of 22.6°.

Example 27. *The a.c. circuit shown in Fig. 7.17, has a choke L and a resistance R. The potential difference across the resistance R is $V_R = 160$ V and that across the choke is $V_L = 120$ V. Find the virtual value of the applied voltage. If the virtual current in the circuit be 1.0 A, then calculate the total impedance of circuit. If the direct current be passed in the circuit, then what will be the potential difference in the circuit ?*

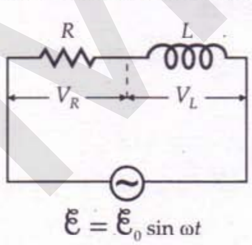


Fig. 7.17

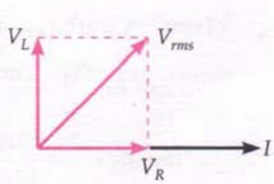


Fig. 7.18

Solution. As V_R is in phase with current I and V_L is 90° ahead of current I in phase, so the phase difference between V_R and V_L is 90°, as shown in Fig. 7.18.

$$\therefore \quad V_{rms} = \sqrt{V_R^2 + V_L^2} = \sqrt{160^2 + 120^2} = \mathbf{200\ V}$$

Impedance, $Z = \frac{V_{rms}}{I_{rms}} = \frac{200}{1.0} = 200\,\Omega$

When direct current ($\omega = 0$) is passed, reactance ωL becomes zero.

$\therefore$ P.D. in the circuit = P.D. across R = **160 V.**

Example 28. *In the circuit shown in Fig. 7.19, the potential difference across the inductor L and resistor R are 120 V and 90 V respectively and the rms value of current is 3 A. Calculate (i) the impedance of the circuit and (ii) the phase angle between the voltage and current.* **[CBSE F 04]**

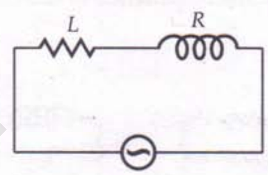


Fig. 7.19

Solution. (*i*) The voltages across *L* and *R* are 90° out of these. Their resultant voltage is

$$V_{rms} = \sqrt{V_R^2 + V_L^2} = \sqrt{90^2 + 120^2} = 150\ \text{V}$$

$$I_{rms} = 3\ \text{A}$$

$\therefore$ Impedance,

$$Z = \frac{V_{rms}}{I_{rms}} = \frac{150}{3} = \mathbf{50\,\Omega}.$$

(*ii*) $\tan\phi = \frac{X_L}{R} = \frac{V_L}{V_R} = \frac{120}{90} = \frac{4}{3}$

$\therefore$ Phase angle, $\phi = \tan^{-1}\frac{4}{3} = \mathbf{53.1°}$.

Problems For Practice

1. Calculate the impedance of a coil of resistance 3 Ω and reactance 4 Ω. (Ans. 5 Ω)
2. An inductance coil has a resistance of 100 ohm. When a.c. signal of frequency 1000 Hz is applied to the coil, the applied voltage leads the current by 45°. Calculate the self-inductance of the coil. **[CBSE D 98C]** (Ans. 0.016 H)

3. An a.c. source of 100 V r.m.s., 50 Hz is connected across a 20 Ω resistor and 2 mH inductor in series. Calculate (*i*) impedance of the circuit and (*ii*) r.m.s. current in the circuit. **[CBSE D 93]**

(Ans. 20 Ω, 5 A)

4. A 100 V, 50 Hz a.c. source is connected to a series combination of an inductance of 100 mH and a resistance of 20 Ω. Calculate the magnitude and phase of the current.

(Ans. 2.68 A, current lags behind voltage by 57.5°)

5. A current of 11 A flows through a coil, when connected to a 110 V d.c. source. When 110 V a.c. of 50 Hz is applied to the coil, only 0.5 A current flows. Calculate the (*i*) resistance (*ii*) impedance and (*iii*) inductance of the coil.

[Ans. (*i*) 10 Ω, (*ii*) 220 Ω, (*iii*) 0.7 H]

6. An arc takes a current of 10 A at 80 V. Find the inductance, which should be put in series to work the arc from 220 V, 50 Hz supply. (Ans. 0.065 H)

7. A current of 2.0 A is flowing in the *LR*-circuit shown in Fig. 7.20. Find the voltage across the a.c. source and the impedance of the circuit.

(Ans. 260 V, 130 Ω)

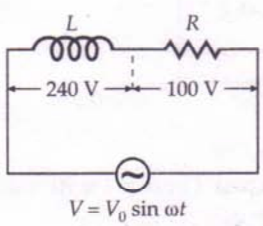


Fig. 7.20

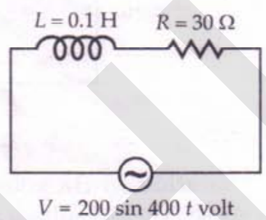


Fig. 7.21

8. Calculate the impedance and the rms current in the a.c. circuit shown in Fig. 7.21. (Ans. 50 Ω, $2\sqrt{2}$ A)

9. The virtual current in the a.c. circuit shown in Fig. 7.22, is 1.0 A. Find (*i*) virtual voltage across the coil *L*, (*ii*) impedance of the circuit and (*iii*) reactance of the coil. [Ans. (*i*) 160 V (*ii*) 200 Ω (*iii*) 160 Ω]

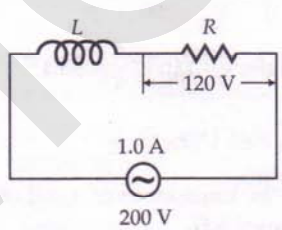


Fig. 7.22

10. A circuit containing a resistance of 50 Ω and an inductance of $(1/\pi)$ H in series is connected to a 200 V a.c. line of frequency 60 Hz. Find the reactance, the impedance, the current in the circuit and the phase difference between the alternating voltage and current.

(Ans. 120 Ω, 130 Ω, 1.538 A, $\tan^{-1} 24$)

11. An a.c. circuit consists of a 220 V, 50 Hz supply connected across a 100 Ω resistor. What inductance should be connected in the circuit in series with resistance so that the current is reduced to half ?

(Ans. 0.55 H)

12. A long solenoid connected to a 12 V d.c. source passes a steady current of 2 A. When the solenoid is connected to an a.c. source of 12 V at 50 Hz, the current flowing is 1 A. Calculate the inductance of the solenoid. (Ans. 33 mH)

13. A choke coil and a resistance are connected in series in an a.c. circuit. A potential difference of 130 V is applied to the circuit. If the potential difference across the resistance is 50 V, what should be the potential difference across the choke coil ?

(Ans. 120 V)

14. An emf, $\mathcal{E} = 200 \sin 377t$ volt is applied across an inductance *L* having a resistance of 1.0 Ω. The maximum current is found to be 10 A. Find the value of *L*. (Ans. 53 mH)

15. An electric circuit containing an inductance *L* and resistance *R* in series has an impedance of 50 Ω at 100 Hz and an impedance of 100 Ω at 500 Hz. Find the values of *L* and *R*. (Ans. 28.13 mH, 46.77 Ω)

16. In the *RL*-circuit shown in Fig. 7.23, resistance $R = 30\ \Omega$, reactance $X_L = 40\ \Omega$ and peak emf = 220 V.

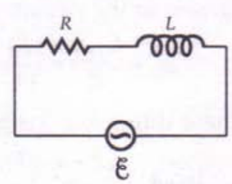


Fig. 7.23

Calculate the (*i*) impedance *Z*, (*ii*) phase difference between the emf and current and (*iii*) the peak current I_0 in the circuit. **[ISCE 03]**

[Ans. (*i*) 50 Ω, (*ii*) 53.1°, (*iii*) 4.4 A]

HINTS

1. $Z = \sqrt{R^2 + X_L^2} = \sqrt{3^2 + 4^2} = \mathbf{5\ \Omega}$.

2. Here $R = 100\ \Omega$, $f = 1000$ Hz, $\phi = 45°$

$$\tan \phi = \frac{X_L}{R} = \frac{2\pi f L}{R}$$

$$\therefore \tan 45° = \frac{2\pi \times 1000 \times L}{100}$$

or $$L = \frac{1 \times 100}{2\pi \times 1000} = \mathbf{0.016\ H.}$$

3. $Z = \sqrt{R^2 + \omega^2 L^2} = \sqrt{R^2 + 4\pi^2 f^2 L^2}$

$= \sqrt{400 + 4 \times 9.87 \times 2500 \times (2 \times 10^{-3})^2} \simeq \mathbf{20\,\Omega}$

$I_{rms} = \frac{\mathcal{E}_{rms}}{Z} = \frac{100}{20} = \mathbf{5\ A}$

4. Impedance,

$Z = \sqrt{R^2 + \omega^2 L^2} = \sqrt{R^2 + 4\pi^2 f^2 L^2}$

$= \sqrt{20^2 + 4\pi^2 \times 50^2 \times (0.1)^2}\ \Omega$

$= \sqrt{400 + 986.96}\ \Omega = \sqrt{1386.96}\ \Omega = 37.24\,\Omega$

$\therefore$ Current, $I = \frac{V_{eff}}{Z} = \frac{100}{37.24}\ \text{A} = \mathbf{2.68\ A}$

Again, $\tan\phi = \frac{\omega L}{R} = \frac{2\pi \times 50 \times 0.1}{20} = 1.5708$

$\therefore \quad \phi = \tan^{-1}(1.5708) \simeq \mathbf{57.5^\circ}$

5. Proceed as in Example 24 on page 7.14.

6. $R = V/I = 80/10 = 8\,\Omega$. When the inductance is put in series, the current should remain same for the new impedance adjusted.

$\therefore \quad I_{eff} = \frac{V_{eff}}{\sqrt{R^2 + 4\pi^2 f^2 L^2}}$

or $\quad 10 = \frac{220}{\sqrt{8^2 + 4\pi^2 \times 50^2 \times L^2}}$

or $\quad 64 + 10000\,\pi^2 L^2 = 484$

or $\quad L^2 = \frac{420}{10000\,\pi^2}$

or $\quad L = \mathbf{0.065\ H}$

7. V_L is 90° ahead of V_R in phase.

$V_{rms} = \sqrt{V_L^2 + V_R^2} = \sqrt{240^2 + 100^2} = \mathbf{260\ V}$

$Z = \frac{V_{rms}}{I_{rms}} = \frac{260}{2.0} = \mathbf{130\,\Omega.}$

8. $V = 220 \sin 400\,t = V_0 \sin \omega t$

$\therefore \quad V_0 = 200\ \text{V},\ \omega = 400\ \text{rad s}^{-1},$

$X_L = \omega L = 400 \times 0.1 = 40\,\Omega$

$\therefore \quad Z = \sqrt{R^2 + X_L^2} = \sqrt{30^2 + 40^2} = \mathbf{50\,\Omega}$

$V_{rms} = \frac{V_0}{\sqrt{2}} = \frac{200}{\sqrt{2}}\ \text{V},$

$I_{rms} = \frac{V_{rms}}{Z} = \frac{200}{\sqrt{2} \times 50} = \mathbf{2\sqrt{2}\ A.}$

9. (*i*) $V_L = \sqrt{V_{rms}^2 - V_R^2} = \sqrt{200^2 - 120^2} = \mathbf{160\ V.}$

(*ii*) $Z = \frac{V_{rms}}{I_{rms}} = \frac{200}{1.0} = \mathbf{200\,\Omega.}$

(*iii*) $X_L = \frac{V_L}{I_{rms}} = \frac{160}{1.0} = \mathbf{160\,\Omega.}$

10. Proceed as in Example 26 on page 7.15.

12. $R = \frac{12\ \text{V}}{2\ \text{A}} = 6\,\Omega$

$Z = \sqrt{R^2 + X_L^2} = \frac{\mathcal{E}_{eff}}{I_{eff}}$

$\therefore \quad \sqrt{R^2 + X_L^2} = \frac{12}{1}$

or $\quad X_L^2 = 144 - R^2 = 144 - 36 = 108$

$X_L = 6\sqrt{3}\,\Omega$

$L = \frac{X_L}{2\pi f} = \frac{6 \times 1.732}{2 \times 3.14 \times 50}$

$= 0.033\ \text{H} = \mathbf{33\ mH.}$

13. $V_L = \sqrt{V_{rms}^2 - V_R^2} = \sqrt{130^2 - 50^2} = \mathbf{120\ V.}$

14. Here $\mathcal{E}_0 = 200\ \text{V},\ \omega = 377\ \text{rad s}^{-1},\ R = 1.0\,\Omega,\ I_0 = 10\text{A}$

$Z = \frac{\mathcal{E}_0}{I_0} = \sqrt{R^2 + X_L^2} \quad \therefore \quad \frac{200}{10} = \sqrt{1^2 + X_L^2}$

$\therefore \quad X_L = \sqrt{20^2 - 1^2} = 19.97\,\Omega;$

$L = \frac{X_L}{\omega} = \frac{19.97}{377} = 0.053\ \text{H} = \mathbf{53\ mH.}$

15. Impedance, $Z = \sqrt{R^2 + 4\pi^2 f^2 L^2}$

At $f = 100$ Hz, $\quad Z = 50\,\Omega$

$\therefore \quad 50 = \sqrt{R^2 + 4\pi^2 (100)^2 L^2}$

or $\quad R^2 + 4\pi^2 (100)^2 L^2 = 2500 \quad \quad ...(i)$

At $f = 500$ Hz, $\quad Z = 100\,\Omega$

$\therefore \quad 100 = \sqrt{R^2 + 4\pi^2 (500)^2 L^2}$

or $R^2 + 4\pi^2 (500)^2 L^2 = 10000 \quad \quad ...(ii)$

On solving equations (*i*) and (*ii*), we get

$L = \mathbf{28.13 \times 10^{-3}\ H}$ and $R = \mathbf{46.77\,\Omega.}$

16. (*i*) $Z = \sqrt{R^2 + X_L^2} = \sqrt{30^2 + 40^2} = \mathbf{50\,\Omega.}$

(*ii*) $\cos\phi = \frac{R}{Z} = \frac{30}{50} = 0.6$

$\therefore \quad \phi = \cos^{-1}(0.6) = \mathbf{53.1^\circ.}$

(*iii*) $I_0 = \frac{\mathcal{E}_0}{Z} = \frac{220}{50} = \mathbf{4.4\ A.}$

7.10 A.C. CIRCUIT WITH RESISTANCE AND CAPACITOR IN SERIES

12. *An alternating emf is applied to a series combination of a resistor and a capacitor. Investigate the phase relationship between current and emf. Find the impedance of the circuit.*

A.C. circuit containing *C* and *R* in series. As shown in Fig. 7.24, consider a resistor *R* and capacitor *C* connected in series to a source of alternating emf $\mathcal{E}$ given by

$$\mathcal{E} = \mathcal{E}_0 \sin \omega t$$

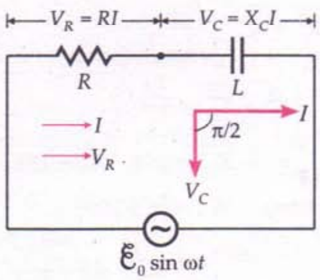


Fig. 7.24 Series *LR*-circuit.

Let I be the current through the series circuit at any instant. Then

1. Voltage $\vec{V}_R = R\vec{I}$ across the resistance R will be *in phase* with the current $\vec{I}$. So phasors $\vec{V}_R$ and $\vec{I}$ are in same direction, as shown in Fig. 7.25. The amplitude of $\vec{V}_R$ is $V_0^R = I_0 R$

2. Voltage $\vec{V}_C = X_C \vec{I}$ across the capacitance C *lags behind* the current $\vec{I}$ in phase by $\pi/2$ rad. So phasor $\vec{V}_C$ lies $\pi/2$ clockwise w.r.t. the phasor $\vec{I}$. Its amplitude is

$$V_0^C = I_0 X_C$$

where X_C is the capacitive reactance.

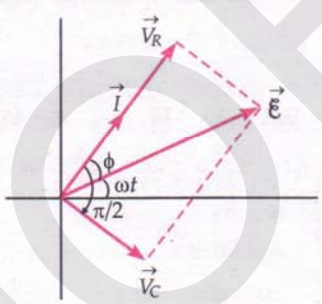


Fig. 7.25 Phasor diagram for a series *CR*-circuit.

By parallelogram law of vector addition,

$$\vec{V}_R + \vec{V}_C = \vec{\mathcal{E}}$$

Using Pythagorean theorem, we get

$$\mathcal{E}_0^2 = (V_0^R)^2 + (V_0^C)^2 = (I_0 R)^2 + (I_0 X_C)^2$$

$$= I_0^2 (R^2 + X_C^2)$$

or $$I_0 = \frac{\mathcal{E}_0}{\sqrt{R^2 + X_C^2}}$$

Clearly, $\sqrt{R^2 + X_C^2}$ is the effective resistance of the series *CR*-circuit which opposes or impedes the flow of current through it and is called its ***impedance*** and is denoted by Z. Thus

$$Z = \sqrt{R^2 + X_C^2} = \sqrt{R^2 + \frac{1}{\omega^2 C^2}} \qquad \left[\because X_C = \frac{1}{\omega C}\right]$$

The phase angle ϕ between the resultant voltage and current is given by

$$\tan \phi = \frac{V_0^C}{V_0^R} = \frac{I_0 X_C}{I_0 R} = \frac{X_C}{R} = \frac{I/\omega C}{R}$$

It is again obvious from the phasor diagram that the current is ahead of emf by phase angle ϕ so the instantaneous value of current is given by

$$I = I_0 \sin(\omega t + \phi)$$

Examples based on Series CR-Circuit

Formulae Used

1. Impedance, $Z = \frac{\mathcal{E}_{rms}}{I_{rms}} = \sqrt{R^2 + X_C^2} = \sqrt{R^2 + \frac{1}{\omega^2 C^2}}$
2. Current, $I_{rms} = \frac{\mathcal{E}_{rms}}{Z}$
3. Phase angle ϕ is given by
$$\tan \phi = \frac{X_C}{R} = \frac{1/\omega C}{R} \quad \text{or} \quad \cos \phi = \frac{R}{Z}$$
4. Instantaneous current, $I = I_0 \sin(\omega t + \phi)$.

Units Used

R, X_C and Z are all in ohm, capacitance C in farad and angular frequency ω in rad s^{-1}.

Example 29. *What is the value of current in the a.c. circuit containing $R = 10\ \Omega$, $C = 50\ \mu C$ in series across 200 V, 50 Hz a.c. source?* [CBSE OD 93]

Solution. Here $R = 10\ \Omega$, $C = 50\ \mu F = 50 \times 10^{-6}$ F, $V_{eff} = 200$ V, $f = 50$ Hz

$$Z = \sqrt{R^2 + \frac{1}{4\pi^2 f^2 C^2}}$$

$$= \sqrt{10^2 + \frac{1}{4\pi^2 \times (50)^2 \times (50 \times 10^{-6})^2}}$$

$$= \sqrt{100 + 4053} = 64.4\ \Omega$$

Current, $$I_{eff} = \frac{V_{eff}}{Z} = \frac{200}{64.4}\ \text{A} = \mathbf{3.10\ A.}$$

Example 30. *When an alternating voltage of 220 V is applied across a device X, a current of 0.5 A flows through the circuit and is in phase with the applied voltage. When the* *same voltage is applied across another device Y, the same current flows through the circuit but it leads the applied voltage by $\pi/2$ radian. (i) Name the devices X and Y. (ii) Calculate the current flowing in the circuit, when same voltage is applied across the series combination of X and Y.* [CBSE OD 97]

Solution. (*i*) Device X is a resistor and Y is a capacitor.

(*ii*) Here $R = X_C = \dfrac{\varepsilon_{eff}}{I_{eff}} = \dfrac{220}{0.5} = 440\ \Omega$

When X and Y are connected in series, their impedance becomes

$$Z = \sqrt{R^2 + X_C^2} = \sqrt{440^2 + 440^2}$$

$$= \sqrt{387200} = 622.25\ \Omega$$

Current, $I_{eff} = \dfrac{\varepsilon_{eff}}{Z} = \dfrac{220}{622.25} = \mathbf{0.35\ A.}$

Example 31. *A series circuit contains a resistor of 20 Ω, a capacitor and an ammeter of negligible resistance. It is connected to a source of 220 V – 50 Hz. If the reading of the ammeter is 2.5 A, calculate the reactance of the capacitor.* [Punjab 99]

Solution. Here $R = 20\ \Omega$, $\varepsilon_{rms} = 220$ V, $f = 50$ Hz, $I_{rms} = 2.5$ A

Impedance, $Z = \dfrac{\varepsilon_{rms}}{I_{rms}} = \dfrac{220}{2.5} = 88\ \Omega$

But $Z = \sqrt{R^2 + X_C^2}$

$\therefore \quad X_C = \sqrt{Z^2 - R^2} = \sqrt{88^2 - 20^2}$

$$= \sqrt{(88+20)(80-20)} = \sqrt{108 \times 68} = \mathbf{85.7\ \Omega}.$$

Example 32. *An alternating current of 1.5 mA rms and angular frequency $\omega = 100$ rad s^{-1} flows through a 10 kΩ resistor and 0.50 μF capacitor in series. Calculate the value of rms voltage across the capacitor and the impedance of the circuit.* [CBSE D 93C]

Solution. Here $\omega = 100$ rad s^{-1},

$$I_{rms} = 1.5\ \text{mA} = 1.5 \times 10^{-3}\ \text{A}$$
$$R = 10\ \text{k}\Omega = 10^4\ \Omega,$$
$$C = 0.50\ \mu\text{F} = 0.5 \times 10^{-6}\ \text{F}$$

Impedance,

$$Z = \sqrt{R^2 + \frac{1}{\omega^2 C^2}}$$

$$= \sqrt{(10^4)^2 + \frac{1}{(100)^2 \times (0.5 \times 10^{-6})^2}}$$

$$= \sqrt{10^8 + 4 \times 10^8} = \sqrt{5 \times 10^8} = \mathbf{2.23 \times 10^4\ \Omega}.$$

The rms voltage across the capacitor is

$$V_{rms}^C = X_C I_{rms} = \frac{1}{\omega C} \times I_{rms}$$

$$= \frac{1}{100 \times 0.5 \times 10^{-6}} \times 1.5 \times 10^{-3}\ \text{V} = \mathbf{30\ V}.$$

Example 33. *A 20 V – 5 W lamp is to run on 200 V – 50 Hz a.c. mains. Find the capacitance of a capacitor required to run the lamp.*

Solution. Current rating of the lamp,

$$I = \frac{P}{V} = \frac{5}{20} = 0.25\ \text{A}$$

Resistance of the lamp,

$$R = \frac{V}{I} = \frac{20}{0.25} = 80\ \Omega$$

In order to run the lamp on 200 V – 50 Hz a.c. mains, a capacitor of capacitance C must be connected in series to increase the effective resistance so that current through the lamp does not exceed 0.25 A. Then

$$Z = \sqrt{R^2 + X_C^2} = \sqrt{R^2 + \left(\frac{1}{2\pi f C}\right)^2} = \sqrt{80^2 + \left(\frac{1}{314\ C}\right)^2}$$

As $\quad I_{rms} = \dfrac{\varepsilon_{rms}}{Z}$

$$\therefore \quad 0.25 = \frac{200}{\sqrt{80^2 + \left(\dfrac{1}{314\ C}\right)^2}}$$

or $\quad 80^2 + \dfrac{1}{(314\ C)^2} = \left(\dfrac{200}{0.25}\right)^2 = 800^2$

or $\quad \dfrac{1}{(314\ C)^2} = 800^2 - 80^2 = 880 \times 720$

or $\quad \dfrac{1}{314\ C} = \sqrt{880 \times 720} = 796$

$$\therefore \quad C = \frac{1}{314 \times 796} = 4.0 \times 10^{-6}\ \text{F} = \mathbf{4.0\ \mu F}.$$

Example 34. *A resistor of 200 Ω and a capacitor of 15.0 μF are connected in series to a 220 V, 50 Hz ac source. (a) Calculate the current in the circuit. (b) Calculate the voltage (rms) across the resistor and the capacitor. Is the algebraic sum of these voltages more than the source voltage? If yes, resolve the paradox.* [NCERT ; CBSE OD 08]

Solution. Here, $R = 200\ \Omega$,

$C = 15.0\ \mu\text{F} = 15.0 \times 10^{-6}$ F, $V_{rms} = 220$ V, $f = 50$ Hz

(*a*) $X_C = \dfrac{1}{2\pi f C} = \dfrac{1}{2 \times 3.14 \times 50 \times 15.0 \times 10^{-6}} = 212.3\ \Omega$

$$Z = \sqrt{R^2 + X_C^2} = \sqrt{(200)^2 + (212.3)^2} = 291.5\ \Omega$$

Therefore, the current in the circuit is

$$I_{rms} = \frac{V_{rms}}{Z} = \frac{220\text{ V}}{291.5\,\Omega} = \mathbf{0.755\ A.}$$

(*b*) As the current is same throughout the series circuit, we have

$$V_{rms}^{R} = I_{rms} \cdot R = 0.755 \times 200 = \mathbf{151\ V}$$

$$V_{rms}^{C} = I_{rms} \cdot X_C = 0.755 \times 212.3 = \mathbf{160.3\ V}$$

The algebraic sum of the two voltages, V_R and V_C is 311.3 V which is more than the source voltage of 220 V. These two voltages are 90° out of phase. These cannot be added like ordinary numbers. The voltage is obtained by using Pythagoras theorem,

$$V_{R+C} = \sqrt{V_R^2 + V_C^2} = \sqrt{(151)^2 + (160.3)^2}$$

$$= 220\text{ V}$$

Thus, if the phase difference between two voltages is properly taken into account, the total voltage across the resistor and the capacitor is equal to the voltage of the source.

Example 35. *In a series R-C circuit, $R = 30\,\Omega$, $C = 0.25\,\mu F$, $V = 100\,V$ and $\omega = 10{,}000\ rad\ s^{-1}$. Find the current in the circuit and calculate the voltage across the resistor and the capacitor.*

Is the algebraic sum of these voltages more than the source voltage ? If yes, resolve the paradox. [CBSE D 04]

Solution. Here $R = 30\,\Omega$, $C = 0.25 \times 10^{-6}$ F, $V_{rms} = 100$ V, $\omega = 10{,}000$ rad s^{-1}

$$X_C = \frac{1}{\omega C} = \frac{1}{10^4 \times 0.25 \times 10^{-6}} = 400\,\Omega$$

$$Z = \sqrt{R^2 + X_C^2}$$

$$= \sqrt{30^2 + 400^2} = \sqrt{160900} = 401.1\Omega$$

$$I_{rms} = \frac{V_{rms}}{Z} = \frac{100}{401.1} \simeq \mathbf{0.25\ A}$$

$$V_{rms}^{R} = I_{rms} \cdot R = 0.25 \times 30 = \mathbf{7.5\ V}$$

$$V_{rms}^{C} = I_{rms} \cdot X_C = 0.25 \times 400 = \mathbf{100\ V}$$

Yes, the algebraic sum of the voltages across *R* and *C* is more than the source voltage of 100 V. This is due to the fact that these voltages are not in the same phase.

Example 36. *An a.c. circuit consists of a series combination of circuit elements 'X' and 'Y'. The current is ahead of the voltage in phase by $\pi/4$. If element 'X' is a pure resistor of $100\,\Omega$, (i) name the circuit element 'Y', and (ii) calculate the rms value of current, if rms value of voltage is 141 V.* [CBSE OD 04C]

Solution. (*i*) The circuit element 'Y' is a capacitor.

(*ii*) Phase angle $\phi = \frac{\pi}{4}$

But $\quad \cos\phi = \frac{R}{Z}$

$\therefore \quad \cos\frac{\pi}{4} = \frac{100\,\Omega}{Z}$

or $\quad Z = \frac{100}{1/\sqrt{2}} = 100\sqrt{2}$

$$= 100 \times 1.414 = 141.4\,\Omega$$

$$I_{rms} = \frac{V_{rms}}{Z} = \frac{141\text{ V}}{141.4\,\Omega} \simeq \mathbf{1\ A.}$$

Problems For Practice

1. A circuit containing a $20\,\Omega$ resistor and $0.1\,\mu$F capacitor in series is connected to 230 V a.c. supply of angular frequency 100 rad s^{-1}. What is the impedance of the circuit ? **[CBSE OD 90]**
(Ans. $10^5\,\Omega$)
2. A circuit consists of a resistance of $10\,\Omega$ and a capacitance of $0.1\,\mu$F. If an alternating emf of 100 V, 50 Hz is applied, find the current in the circuit. **[Haryana 92]**
(Ans. 3.14 mA)
3. A 20 W, 50 V, filament is connected in series to an a.c. mains of 250 V, 50 Hz. Calculate the value of the capacitor required to run the lamp. (Ans
4. Find the impedance of the circuit shown in Fig. 7.26, for (*i*) direct current and (*ii*) alternating current of frequency $10/\pi$ kHz.
[Ans. (*i*) Infinite (*ii*) $32\,\Omega$]

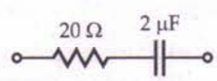


Fig. 7.26

5. A $1\,\mu$F capacitor is connected to a 220 V – 50 Hz a.c. source. Find the virtual value of current through circuit. What is the peak value of voltage across the capacitor ? **[CBSE OD 2000 C]**
(Ans. 0.07 A, 311 V)
6. A capacitor in series with a resistance of $30\,\Omega$ is connected to a.c. mains. The reactance of the capacitor is $40\,\Omega$. Calculate the phase difference between the current and the supply voltage.
(Ans. $\tan^{-1}\frac{4}{3}$)
7. A circuit has a resistance of $10\,\Omega$ and its impedance is $100\sqrt{2}\,\Omega$. Find the reactance of the circuit.
(Ans. $100\,\Omega$)

HINTS

1. Here $R = 20\,\Omega$, $C = 0.1\,\mu F = 0.1 \times 10^{-6}\,F = 10^{-7}\,F$, $\omega = 100$ rad s^{-1}

Impedance,

$$Z = \sqrt{R^2 + \frac{1}{\omega^2 C^2}}$$

$$= \sqrt{20^2 + \frac{1}{(100)^2 \times (10^{-7})^2}}$$

$$= \sqrt{400 + 10^{10}}\ \Omega \simeq \mathbf{10^5\ \Omega}.$$

2. $X_C = \frac{1}{2\pi fC} = \frac{1}{2 \times 3.14 \times 50 \times 0.1 \times 10^{-6}} = 3.2 \times 10^4\,\Omega$

$$Z = \sqrt{R^2 + X_C^2} = \sqrt{100 + 10.24 \times 10^8} = 3.2 \times 10^4\,\Omega$$

$$I_{rms} = \frac{\mathcal{E}_{rms}}{Z} = \frac{100}{3.2 \times 10^4} = 3.14 \times 10^{-3}\,A = \mathbf{3.14\ mA}$$

3. $I_{rms} = \frac{20}{50} = 0.4\,A, \quad R = \frac{50}{0.4} = 125\,\Omega$

$$Z = \frac{250}{0.4} = 625\,\Omega$$

$$X_C = \sqrt{Z^2 - R^2} = \sqrt{625^2 - 125^2}$$

$$= \sqrt{750 \times 500} = 612.37$$

or $\quad \frac{1}{2\pi fC} = 612.37$

$$C = \frac{1}{2 \times 3.14 \times 50 \times 612.37}$$

$$= 5.2 \times 10^{-6}\,F = \mathbf{5.2\,\mu F}.$$

5. $X_C = \frac{1}{2\pi\, f\, C} = \frac{1}{2 \times 3.14 \times 50 \times 10^{-6}} = 3.18 \times 10^3\,\Omega$

$$I_{rms} = \frac{V_{rms}}{X_C} = \frac{220}{3.18 \times 10^3} = \mathbf{0.07\ A.}$$

$$V_0 = \sqrt{2}\, V_{rms} = 1.414 \times 220 = \mathbf{311\ V.}$$

6. $\tan\phi = \frac{X_C}{R} = \frac{40}{30} = \frac{4}{5}$

$\therefore \quad \phi = \tan^{-1}\frac{4}{3}.$

7. Reactance, $X = \sqrt{Z^2 - R^2}$.

7.11 SERIES *LCR*-CIRCUIT

13. *Using phasor diagram, derive an expression for the impedance of a series LCR-circuit.*

Series *LCR*-circuit. As shown in Fig. 7.27, suppose a resistance *R*, an inductance *L* and capacitance *C* are connected in series to a source of alternating emf $\mathcal{E}$ given by

$$\mathcal{E} = \mathcal{E}_0 \sin \omega t$$

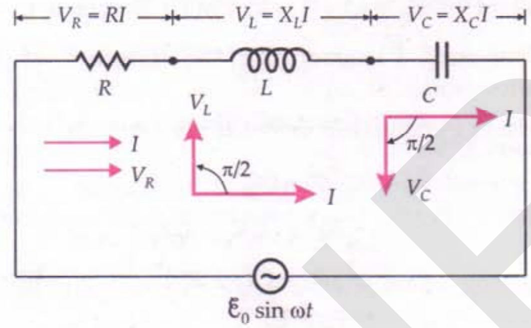


Fig. 7.27 A series *LCR*-circuit.

Let *I* be the current in the series circuit at any instant. Then

1. Voltage $\vec{V}_R = R\vec{I}$ across the resistance *R* will be *in phase* with current $\vec{I}$. So phasors $\vec{V}_R$ and $\vec{I}$ are in same direction, as shown in Fig. 7.28. The amplitude of $\vec{V}_R$ is

$$V_0^R = I_0 R$$

2. Voltage $\vec{V}_L = X_L \vec{I}$ across the inductance *L* is *ahead* of current *I* in phase by $\pi/2$ rad. So phasor $\vec{V}_L$ lies $\pi/2$ rad anticlockwise w.r.t. the phasor $\vec{I}$. Its amplitude is

$$V_0^L = I_0 X_L$$

3. Voltage $\vec{V}_C = X_C \vec{I}$ across the capacitance *C lags behind* the current $\vec{I}$ in phase by $\pi/2$ rad. So phasor $\vec{V}_C$ lies $\pi/2$ clockwise w.r.t. the phasor $\vec{I}$. Its amplitude is

$$V_0^C = I_0\, X_C$$

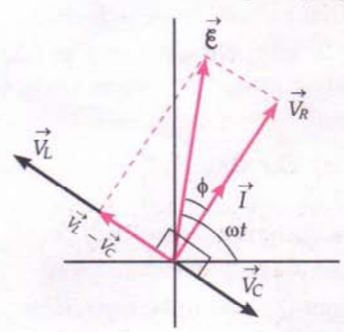


Fig. 7.28 Phasor diagram for a series LCR-circuit when $V_L > V_C$.

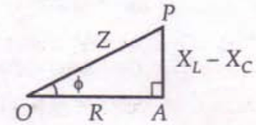


Fig. 7.29 Impedance triangle when $X_L > X_C$.

As $\vec{V}_L$ and $\vec{V}_C$ are in opposite directions, their resultant is $(\vec{V}_L - \vec{V}_C)$. By parallelogram law, the resultant of $\vec{V_R}$ and $(\vec{V_L} - \vec{V_C})$ must be equal to the applied emf $\vec{\varepsilon}$, given by the diagonal of the parallelogram.

Using Pythagorean theorem, we get

$$\varepsilon_0^2 = (V_0^R)^2 + (V_0^L - V_0^C)^2$$
$$= (I_0R)^2 + (I_0X_L - I_0X_C)^2$$
$$= I_0^2 [R^2 + (X_L - X_C)^2]$$

or $$I_0 = \frac{\varepsilon_0}{\sqrt{R^2 + (X_L - X_C)^2}}$$

Clearly, $\sqrt{R^2 + (X_L - X_C)^2}$ is the *effective resistance of the series LCR-circuit which opposes or impedes the flow of current through it and is called its* ***impedance***. It is denoted by Z and its SI unit is ohm (Ω). Thus

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

The relationship between the resistance R, inductive reactance X_L, capacitive reactance X_C and the impedance Z is shown in Fig. 7.29. The right angled Δ *OAP* is called the ***impedance triangle***.

Special Cases

1. When $X_L > X_C$ or $V_L > V_C$, we see from Fig. 7.28 that emf is ahead of current by phase angle ϕ which is given by

$$\tan \phi = \frac{X_L - X_C}{R} \quad \text{or} \quad \cos \phi = \frac{R}{Z}$$

The instantaneous current in the circuit will be

$$I = I_0 \sin (\omega t - \phi)$$

The series *LCR*-circuit is said to be ***inductive***.

2. When $X_L < X_C$ or $V_L < V_C$, we see from Fig. 7.30 that current is ahead of emf by phase angle ϕ which is given by

$$\tan \phi = \frac{X_C - X_L}{R} \quad \text{or} \quad \cos \phi = \frac{R}{Z}$$

The instantaneous current in circuit will be

$$I = I_0 \sin (\omega t + \phi)$$

The series *LCR*-circuit is said to be ***capacitive***.

3. When $X_L = X_C$ or $V_L = V_C$, $\phi = 0$, the emf and current will be in the same phase. The series *LCR*-circuit said to be ***purely resistive***.

It may also be noted that

$$I_0 = \frac{\varepsilon_0}{Z} \quad \text{or} \quad \frac{I_0}{\sqrt{2}} = \frac{\varepsilon_0}{\sqrt{2}Z} \quad \text{or} \quad I_{rms} = \frac{\varepsilon_{rms}}{Z}$$

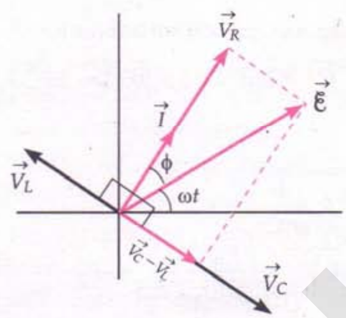


Fig. 7.30 Phasor diagram for a series *LCR*-circuit when $V_L < V_C$.

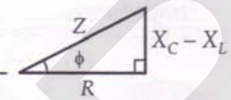


Fig. 7.31 Impedance triangle when $X_L < X_C$.

14. *Define the terms susceptance and admittance. Give their units.*

Susceptance. *The reciprocal of the reactance of an a.c. circuit is called its susceptance.* Its SI unit is ohm^{-1} or mho.

Admittance. *The reciprocal of the impedance of an a.c. circuit is called its admittance.* Its SI unit is ohm^{-1} or mho.

7.12 RESONANCE CONDITION OF A SERIES *LCR*-CIRCUIT

15. *What do you mean by the resonance condition of a series LCR-circuit ? Calculate its resonant frequency.*

Resonance condition of a series *LCR*-circuit. *A series LCR-circuit is said to be in the resonance condition when the current through it has its maximum value.*

The current amplitude I_0 for a series *LCR*-circuit is given by

$$I_0 = \frac{\varepsilon_0}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

Clearly, I_0 becomes zero both for $\omega \to 0$ and $\omega \to \infty$. The value of I_0 is maximum when

$$\omega L - \frac{1}{\omega C} = 0 \quad \text{or} \quad \omega = \frac{1}{\sqrt{LC}}$$

Then impedance, $Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} = R$

Clearly the impedance is minimum. The circuit is purely resistive. The current and voltage are in the same phase and the current in the circuit is maximum. This condition of the *LCR*-circuit is called *resonance condition*. The frequency at which the current amplitude I_0 attains a peak value is called ***natural*** or ***resonant frequency*** of the *LCR*-circuit and is denoted by f_r.

Thus $\quad \omega_r = 2\pi f_r = \frac{1}{\sqrt{LC}} \quad$ or $\quad f_r = \frac{1}{2\pi\sqrt{LC}}$

The current amplitude at resonant frequency will be

$$I_0 = \frac{\varepsilon_0}{R}.$$

16. *Give some of the important characteristics of the series resonant circuit.*

Characteristics of series resonant circuit :

1. Resonance occurs in a series *LCR*-circuit when $X_L = X_C$.
2. Resonant frequency, $f_r = \dfrac{1}{2\pi\sqrt{LC}}$.
3. The impedance is minimum and purely resistive.
4. The current has a maximum value of $(\mathcal{E}_0 / R)$ at resonant condition.
5. The power dissipated in the circuit is maximum and is equal to $\mathcal{E}_{rms}^2 / R$.
6. The current is in phase with the voltage or the power factor is unity ($\cos\phi = 1$ when $\phi = 0$).
7. Series resonance can occur at all values of resistance *R*.
8. The voltage across *R* is equal to the applied emf.
9. The voltages across *L* and *C* are equal and have a phase difference of 180° and so their resultant is zero.
10. The voltages across *L* and *C* are very high as compared to the applied voltage. Hence a series *LCR*-circuit is used to obtain a large magnification of a.c. voltage.
11. The series resonant circuit is also called an ***acceptor circuit***. When a number of frequencies are fed to it, it accepts only one frequency f_r and rejects the other frequencies. The current is maximum for this frequency.

For Your Knowledge

➢ Resonance occurs in a series *LCR*-circuit when $X_L = X_C$ or $\omega_r = 1/\sqrt{LC}$. For resonance to occur, the presence of both *L* and *C* elements in the circuit is essential. Only then the voltages *L* and *C* (being 180° out of phase) will cancel each other and current amplitude will be $\mathcal{E}_0 / R$ *i.e.*, the total source voltage will appear across *R*. So we cannot have resonance in *LR*-and *LC*-circuits.

7.13 SHARPNESS OF RESONANCE : Q-FACTOR

17. *What do you mean by sharpness of resonance in a series resonant circuit ? Find expression for Q-factor of the circuit.*

Sharpness of resonance : Q-Factor. Fig. 7.32 shows the variation of current amplitude I_0 in a series *LCR*-circuit with angular frequency ω, for three different values of *R*. The current amplitude has a peak at the resonant frequency $\omega_r = \dfrac{1}{\sqrt{LC}}$ and falls to zero in either direction. The resonant frequency is independent of *R*, but the sharpness of peak depends on *R*. The peak is higher for smaller values of *R*. Thus the resonance is sharp for small *R* and a flat one for large *R*. The sharpness of resonance is measured by a coefficient called the ***quality*** or ***Q-factor*** of the circuit.

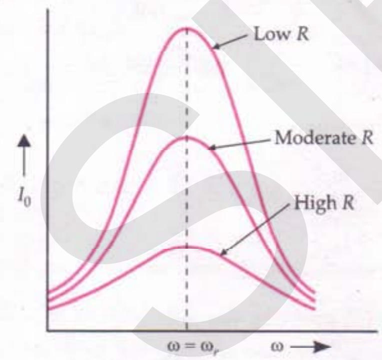


Fig. 7.32 Variation of current amplitude with frequency in an *LCR*-circuit.

The Q-factor of a ***series resonant*** *circuit is defined as the ratio of the resonant frequency to the difference in two frequencies taken on both sides of the resonant frequency such that at each frequency, the current amplitude becomes* $\dfrac{1}{\sqrt{2}}$ *times the value at resonant frequency.*

Mathematically, the Q-factor can be expressed as

$$Q = \frac{\omega_r}{\omega_2 - \omega_1} = \frac{\omega_r}{2\,\Delta\omega} = \frac{\text{Resonant frequency}}{\text{Bandwidth}}$$

where ω_1 and ω_2 are the frequencies at which the current falls to $\dfrac{1}{\sqrt{2}}$ times its resonant value, as shown in Fig. 7.33 and we have used

$$\omega_1 = \omega_r - \Delta\omega \;; \quad \omega_2 = \omega_r + \Delta\omega$$

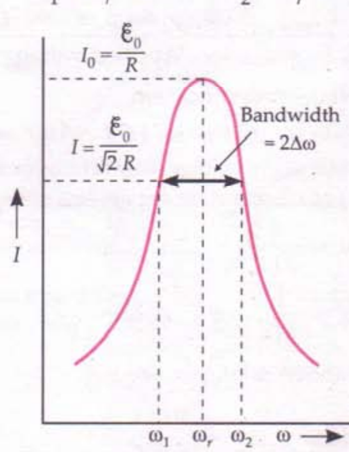


Fig. 7.33 Bandwidth of a series resonant circuit.

The frequency range $\omega_2 - \omega_1 = 2\Delta\omega$ is called ***bandwidth***. The larger the value of Q-factor, the smaller is the value of $2\Delta\omega$ or the bandwidth and sharper is the peak in the current. Q-factor is a *dimensionless quantity*.

Expression for Q-factor. Clearly at ω_r, the impedance is equal to R, while at ω_1 and ω_2 its value is $\sqrt{2}\ R$.

$$\therefore \quad Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} = \sqrt{2}\ R$$

$$\text{or} \quad R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2 = 2R^2 \quad \text{or} \quad \omega L - \frac{1}{\omega C} = \pm R$$

We can write

$$\omega_1 L - \frac{1}{\omega_1 C} = -R \qquad ...(1)$$

and

$$\omega_2 L - \frac{1}{\omega_2 C} = +R \qquad ...(2)$$

Adding (1) and (2), we get

$$(\omega_1 + \omega_2)L - \frac{1}{C}\left(\frac{\omega_1 + \omega_2}{\omega_1\ \omega_2}\right) = 0$$

or

$$\omega_1\ \omega_2 = \frac{1}{LC}$$

Subtracting (1) from (2), we get

$$(\omega_2 - \omega_1)\ L + \frac{1}{C}\left(\frac{\omega_2 - \omega_1}{\omega_1 \omega_2}\right) = 2\ R$$

or

$$(\omega_2 - \omega_1)\left(L + \frac{1}{C\omega_1\omega_2}\right) = 2\ R$$

or

$$(\omega_2 - \omega_1)(L + L) = 2\ R \quad \left[\because \omega_1\omega_2 = \frac{1}{LC}\right]$$

or

$$\omega_2 - \omega_1 = \frac{R}{L}$$

$$\therefore \quad Q = \frac{\omega_r}{2\Delta\omega} = \frac{\omega_r}{\omega_1 - \omega_2} = \frac{\omega_r L}{R} \qquad ...(3)$$

The above equation can also be written as

$$Q = \frac{\omega_r\ L\ I_{rms}}{R\ I_{rms}} = \frac{\text{Voltage drop across } L(\text{ or } C)}{\text{Applied voltage}}$$

= Voltage magnification

Thus the Q-factor of a series LCR-circuit may also be defined as the ratio of the voltage drop across the inductance (or capacitance) at resonance to the applied voltage.

$$\text{As} \quad \omega_r = \frac{1}{\sqrt{LC}}$$

$$\therefore \quad \omega_r^2 = \frac{1}{LC} \quad \text{or} \quad \omega_r L = \frac{1}{\omega_r C}$$

Using the above relations, we get

$$Q = \frac{1}{\omega_r\ CR} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

From equation (3), we see that if Q-factor is large *i.e.*, if R is low or L is large, the bandwidth $2\Delta\omega$ is small. This means that the resonance is sharp or the series resonant circuit is more selective.

18. *Describe the use of a series resonant circuit in the tuning of a radio receiver.*

Tuning of a radio receiver. The tuning circuit of a radio or TV is an example of LCR resonant circuit. Signals are transmitted by different stations at different frequencies. These frequencies are picked up by the antenna and corresponding to these frequencies, a number of voltages appear across the series LCR-circuit. But maximum current flows through the circuit for that a.c. voltage which has frequency equal to $f_r = \frac{1}{2\pi\sqrt{LC}}$. If Q-value of the circuit is large, the signals of the other stations will be very weak. By changing the value of the adjustable capacitor C, the signal from the desired station can be tuned in.

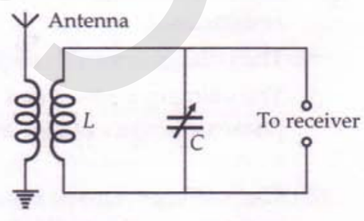


Fig. 7.34 Series resonant circuit.

Examples based on Series LCR-Circuit, its Resonance and Q-Factor

Formulae Used

1. Impedance of a series LCR-circuit,
$$Z = \frac{\varepsilon_{rms}}{I_{rms}} = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$
2. Phase angle ϕ between current and voltage is given by
$$\tan\phi = \frac{X_L - X_C}{R} \quad \text{or} \quad \cos\phi = \frac{R}{Z}$$
3. Resonant frequency of LCR-series circuit (when $X_L = X_C$),
$$f_r = \frac{1}{2\pi\sqrt{LC}}$$
4. $$\text{Q-Factor} = \frac{\omega_r}{\omega_2 - \omega_1} = \frac{\omega_r\ L}{R} = \frac{1}{R}\sqrt{\frac{L}{C}}$$
where ω_1 and ω_2 are the frequencies at which current falls to $1/\sqrt{2}$ times its resonant value.

Units Used

R, X_L, X_C and Z are all in ohm, inductance L in henry, capacitance C in farad, angular frequencies ω_1, ω_2 and ω_r in rad s^{-1}.

Example 37. *Determine the impedance of a series LCR-circuit if the reactance of C and L are 250 Ω and 220 Ω respectively and R is 40 Ω.* [CBSE F 94C]

Solution. Here $X_C = 250\,\Omega$, $X_L = 220\,\Omega$, $R = 40\,\Omega$

Impedance,

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$= \sqrt{40^2 + (220-250)^2} = \sqrt{1600+900} = \mathbf{50\,\Omega}.$$

Example 38. *A resistor of 50 ohm, an inductor of $(20/\pi)$ H and a capacitor of $(5/\pi)\,\mu F$ are connected in series to a voltage source 230 V, 50 Hz. Find the impedance of the circuit.* [CBSE OD 06]

Solution. Here $R = 50\,\Omega$, $L = \frac{20}{\pi}$ H,

$C = \frac{5}{\pi}\,\mu\text{ F} = \frac{5}{\pi}\times 10^{-6}\text{F}$, $\mathcal{E}_{eff} = 230$ V, $f = 50$ Hz

$$X_L = 2\pi f\,L = \frac{20}{\pi}\times 2\times\pi\times 50 = 2000\,\Omega$$

$$X_C = \frac{1}{C\times 2\pi f} = \frac{1}{(5/\pi)\times 10^{-6}\times 2\times\pi\times 50} = 2000\,\Omega$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$= \sqrt{(50)^2 + (2000-2000)^2}$$

$$= \sqrt{2500}\,\Omega = \mathbf{50\,\Omega}.$$

Example 39. *What will be the readings in the voltmeter and ammeter of the circuit shown in Fig. 7.35 ?*

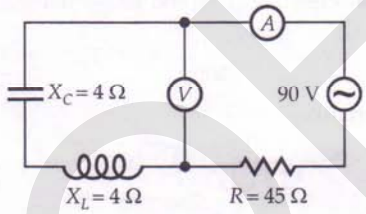


Fig. 7.35

Solution. Impedance of the circuit is

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{45^2 + (4-4)^2} = 45\,\Omega$$

Reading of the ammeter $= I_{rms} = \frac{\mathcal{E}_{rms}}{Z} = \frac{90}{45} = \mathbf{2\ A}$

Reading of the voltmeter

$$= (X_L - X_C)\,I_{rms} = (4-4)\times 2 = \mathbf{0}.$$

Example 40. *A 0.3 H inductor, 60 μF capacitor and a 50 Ω resistor are connected in series with a 120 V, 60 Hz supply. Calculate*

(*i*) *impedance of the circuit*

(*ii*) *current flowing in the circuit.* [CBSE F 94]

Solution. Here $L = 0.3$ H, $C = 60\,\mu\text{F} = 60\times 10^{-6}$ F,

$R = 50\,\Omega$, $V_{eff} = 120$ V, $f = 60$ Hz

(*i*) Inductive reactance,

$$X_L = 2\pi\,f\,L = 2\times 3.14\times 60\times 0.3 = 113.04\,\Omega$$

Capacitive reactance,

$$X_C = \frac{1}{2\pi\,f\,C} = \frac{1}{2\times 3.14\times 60\times 60\times 10^{-6}}$$

$$= 44.23\,\Omega$$

Net reactance

$$= X_L - X_C = 113.04 - 44.23 = 68.81\,\Omega$$

Impedance,

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{50^2 + (68.81)^2}$$

$$= \sqrt{2500 + 4734.8} = \sqrt{7234.8} \simeq \mathbf{85}\ \Omega.$$

(*ii*) Current in the circuit is

$$I_{eff} = \frac{V_{eff}}{Z} = \frac{120}{85} = \mathbf{1.41\ A.}$$

Example 41. *A resistor of 12 ohm, a capacitor of reactance 14 ohm and a pure inductor of inductance 0.1 henry are joined in series and placed across a 200 volt, 50 Hz a.c. supply. Calculate*

(*i*) *the current in the circuit and*

(*ii*) *the phase angle between the current and the voltage. Take $\pi = 3$ for purpose of calculations.*

Solution. Here $R = 12\,\Omega$, $X_C = 14\,\Omega$, $L = 0.1$ H

$$\mathcal{E}_{eff} = 200\text{ V},\ f = 50\text{ Hz}$$

Impedance,

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

and $\quad X_L = \omega L = 2\pi f\,L = 2\times 3\times 50\times 0.1 = 30\,\Omega$

$\therefore \quad Z = \sqrt{(12)^2 + (30-14)^2} = \sqrt{144+256}\,\Omega$

or $\quad Z = \sqrt{400} = 20\,\Omega.$

(*i*) The current in the circuit,

$$I_{eff} = \frac{\mathcal{E}_{eff}}{Z} = \frac{200}{20} = \mathbf{10\ A.}$$

(*ii*) The phase angle ϕ between the current and voltage is given by

$$\tan\phi = \frac{X_L - X_C}{R} = \frac{30-14}{12} = \frac{16}{12} = \frac{4}{3} = 1.3333$$

$\therefore \quad \phi = \tan^{-1}(1.3333) \simeq \mathbf{53.1°}.$

Example 42. *Figure 7.36 shows series LCR circuit with $L = 5.0$ H, $C = 80\,\mu$F, $R = 40\Omega$ connected to a variable frequency 240 V source. Calculate :*

(*i*) *the angular frequency of the source which drives the circuit at resonance.*

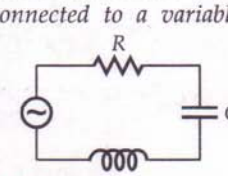


Fig. 7.36

(ii) the current at the resonating frequency.

(iii) the rms potential drop across the capacitor at resonance. [CBSE D 12]

Solution. Here $V_{rms} = 240$ V, $L = 5.0$ H,

$C = 80\ \mu F = 80 \times 10^{-6}$ F, $R = 40\ \Omega$

(i) $\omega_r = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{5 \times 80 \times 10^{-6}}} = \mathbf{50\ rad\ s^{-1}}$.

(ii) At resonance, $Z = R = 40\ \Omega$

$$I_{rms} = \frac{V_{rms}}{Z} = \frac{240}{40} = \mathbf{6\ A}$$

(iii) $V_{rms}^C = I_{rms} \times \frac{1}{\omega_r C} = 6 \times \frac{1}{50 \times 80 \times 10^{-6}} = \mathbf{1500\ V}$.

Example 43. *A resistor of resistance 400 Ω, and a capacitor of reactance 200 Ω, are connected in series to a 220 V, 50 Hz, a.c. source. If the current in the circuit is 0.49 ampere, find the (i) voltage across the resistor and capacitor (ii) value of inductance required so that voltage and current are in phase.* [CBSE Sample Paper 11]

Solution. Here $R = 400\ \Omega$, $X_C = 200\ \Omega$,

$f = 50$ Hz, $I_{rms} = 0.49$ A

(i) $V_{rms}^R = I_{rms} \cdot R = 0.49 \times 400 = \mathbf{196\ V}$

$V_{rms}^C = I_{rms} \cdot X_C = 0.49 \times 200 = \mathbf{98\ V}$.

(ii) As V and I are in same phase,

$$X_L = X_C \quad \text{or} \quad 2\pi f L = X_C$$

$$\therefore \quad L = \frac{X_C}{2\pi f} = \frac{200 \times 7}{2 \times 22 \times 50} = \mathbf{0.64\ H.}$$

Example 44. *A resistance of 2 ohms, a coil of inductance 0.01 H are connected in series with a capacitor, and put across a 200 volt, 50 Hz supply. Calculate :*

(i) the capacitance of the capacitor so that the circuit resonates.

(ii) the current and voltage across the capacitor at resonance. (Take π = 3) [CBSE D 99C]

Solution. Here $R = 2\ \Omega$, $L = 0.01$ H, $\mathcal{E}_{eff} = 200$ V, $f = 50$ Hz

(i) Resonance frequency,

$$f = \frac{1}{2\pi\sqrt{LC}}$$

$$\therefore \quad C = \frac{1}{4\pi^2 f^2 L} = \frac{1}{4 \times (3)^2 \times (50)^2 \times (0.01)}$$

$$= \frac{1}{4 \times 9 \times 2500 \times 0.01} = \frac{1}{900}$$

$$= 0.0011\ F = \mathbf{11 \times 10^{-4}\ F.}$$

(ii) $I_{eff} = \frac{\mathcal{E}_{eff}}{R} = \frac{200}{2} = \mathbf{100\ A}$

$$\therefore \quad V_C = I_{eff} X_C = I_{eff} \frac{1}{2\pi f C}$$

$$= \frac{100}{2 \times 3 \times 50 \times 11 \times 10^{-4}}$$

$$= \frac{100 \times 10^4}{3300} = \mathbf{303.03\ V.}$$

Example 45. *An inductor coil joined to a 6 V battery draws a steady current of 12 A. This coil is connected in series to a capacitor and a.c. source of alternating emf 6 V. If the current in the circuit is in phase with the emf, find the rms current.*

Solution. Resistance of the coil,

$$R = \frac{V}{I} = \frac{6}{12} = 0.5\ \Omega$$

In the a.c. circuit, the current is in phase with the emf.

∴ Impedance,

$$Z = R = 0.5\ \Omega, \quad I_{rms} = \frac{\mathcal{E}_{rms}}{Z} = \frac{6}{0.5} = \mathbf{12\ A.}$$

Example 46. *A radio wave of wavelength 300 m can be transmitted by a transmission centre. A condenser of capacity 2.4 μF is available. Calculate the inductance of the required coil for resonance.*

Solution. Frequency of the radio wave,

$$\nu = \frac{c}{\lambda} = \frac{3 \times 10^8}{300} = 10^6\ \text{Hz}$$

For resonance,

$$\nu = \frac{1}{2\pi\sqrt{LC}} \quad \text{or} \quad \nu^2 = \frac{1}{4\pi^2 LC}$$

∴ Inductance,

$$L = \frac{1}{4\pi^2 \nu^2 C}$$

$$= \frac{1}{4 \times 9.87 \times (10^6)^2 \times 2.4 \times 10^{-6}}$$

$$= \mathbf{1.055 \times 10^8\ H.}$$

Example 47. *A 25.0 μF capacitor, a 0.10 henry inductor and a 25.0 ohm resistor are connected in series with an A.C. source whose emf is given by* $\mathcal{E} = 310 \sin 314\, t$ *(volt).*

(i) What is the frequency of the emf ?

(ii) What is the reactance of the circuit ?

(iii) What is the impedance of the circuit ?

(iv) What is the current of the circuit ?

(v) What is the phase angle of the current by which it leads or lags the applied emf ?

(*vi*) *What is the expression for the instantaneous value of current in the circuit ?*

(*vii*) *What are the effective voltages across the capacitor, the inductor and the resistor ?*

(*viii*) *Construct a vector diagram for these voltages.*

(*ix*) *What value of inductance will make the impedance of circuit minimum ?* [CBSE OD 95, upto part (*iv*)]

Solution. (*i*) Given $\mathcal{E} = 310 \sin 314\, t$ (volt)

Comparing it with $\mathcal{E} = \mathcal{E}_0 \sin 2\pi ft$, we get

$$2\pi f = 314 \text{ or } f = \frac{314}{2\pi} = \frac{314}{2 \times 3.14} = \mathbf{50\ Hz.}$$

(*ii*) $$X_C = \frac{1}{2\pi f C} = \frac{1 \times 7}{2 \times 22 \times 50 \times 25 \times 10^{-6}} = 127.3\,\Omega$$

$[\because 1\,\mu F = 10^{-6} F]$

$$X_L = 2\pi f L = 2 \times \frac{22}{7} \times 50 \times 0.1 = 31.4\,\Omega$$

As X_L and X_C are out of phase by 180°, therefore,

Net reactance

$$= X_C - X_L = 127.3 - 31.4 = \mathbf{95.9\ \Omega}$$

and it is capacitive.

(*iii*) Impedance,

$$Z = \sqrt{R^2 + (X_C - X_L)^2} = \sqrt{(25)^2 + (95.9)^2}$$
$$= \sqrt{625 + 9196.81} = \sqrt{9821.81} = \mathbf{99.1\ \Omega.}$$

(*iv*) Effective current,

$$I_{eff} = \frac{\mathcal{E}_{eff}}{Z}$$

But $$\mathcal{E}_{eff} = \frac{\mathcal{E}_0}{\sqrt{2}} = \frac{310}{\sqrt{2}} \simeq 220\text{ V}$$

$\therefore$ $$I_{eff} = \frac{220}{99.1} = \mathbf{2.22\ A.}$$

(*v*) The phase angle ϕ is given by

$$\tan\phi = \frac{X_C - X_L}{R} = \frac{95.9}{25} = 3.84$$

Hence $\phi = 75.4°$ or **1.31 rad.**

As the circuit is capacitive, the current leads the voltage by 75.4°.

(*vi*) The instantaneous current is given by

$$I = I_0 \sin(2\pi ft + \phi)$$

But $$I_0 = I_{eff}\sqrt{2} = 2.22\sqrt{2} = 3.13\text{ A}$$

$\therefore$ $$I = \mathbf{3.13 \sin(314t + 1.31).}$$

(*vii*) Effective voltage across the capacitor is

$$V_C = I_{eff}\, X_C = 2.22 \times 127.3 = \mathbf{282.6\ V}$$

Effective voltage across the inductor is

$$V_L = I_{eff}\, X_L = 2.22 \times 31.4 = \mathbf{69.7\ V}$$

Effective voltage across the resistor is

$$V_R = I_{eff}\, R = 2.22 \times 25 = \mathbf{55.5\ V.}$$

(*viii*) Vector diagram of voltages is shown in Fig. 7.37.

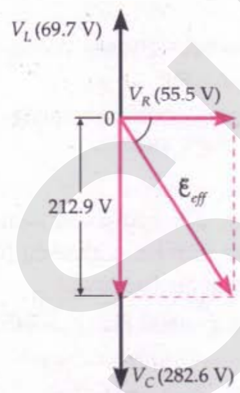


Fig. 7.37

(*ix*) Impedance, $$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

Z is minimum, if $X_L = X_C$

or if $$2\pi f L = \frac{1}{2\pi f C}$$

or $$L = \frac{1}{4\pi^2 f^2 C} = \frac{7 \times 7}{4(22)^2 (50)^2 \times 25 \times 10^{-6}}$$
$$= \mathbf{0.405\ H.}$$

Example 48. *A 2 μF capacitor, 100 Ω resistor and 8 H inductor are connected in series with an a.c. source. What should be the frequency of the a.c. source, for which the current drawn in the circuit is maximum ? If the peak value of emf of the source is 200 V, find for maximum current :*

(*i*) *the inductive and capacitive reactances of the circuit,*

(*ii*) *total impedance of the circuit,*

(*iii*) *peak value of current in the circuit,*

(*iv*) *the phase difference between voltages across inductor and resistor, and*

(*v*) *the phase difference between voltages across inductor and capacitor.* [ISCE 98]

Solution. Here $C = 2\,\mu F = 2 \times 10^{-6}$ F, $R = 100\,\Omega$, $L = 8$ H, $\mathcal{E}_0 = 200$ V

The current drawn in the circuit will be maximum when the frequency of the a.c. source is equal to the resonant frequency f_r of the circuit.

$$f_r = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{8 \times 2 \times 10^{-6}}} = \frac{10^3}{8\pi} = \mathbf{39.8\ Hz.}$$

(*i*) X_L or X_C at resonant frequency,

$$= 2\pi f_r L = 2\pi \times \frac{10^3}{8\pi} = \mathbf{2000\,\Omega}.$$

(*ii*) Total impedance at resonance, $Z = R = \mathbf{100\,\Omega}$.

(*iii*) Peak value of current,

$$I_0 = \frac{\mathcal{E}_0}{Z} = \frac{\mathcal{E}_0}{R} = \frac{200}{100} = \mathbf{2\ A}.$$

(*iv*) Phase difference between voltages across inductor and resistor = **90°**.

(*v*) Phase difference between voltages across inductor and capacitor = **180°**.

Example 49. *In a series LCR-circuit, the resonant frequency is 800 Hz. The half power points are obtained at frequencies 745 and 855 Hz. Calculate the Q-factor of the circuit. Also calculate the bandwidth.*

Solution. Here $f_r = 800$ Hz, $f_1 = 745$ Hz, $f_2 = 855$ Hz

(*i*) $Q = \dfrac{f_r}{f_2 - f_1} = \dfrac{800}{855 - 745} = \dfrac{800}{110} = \mathbf{7.27}.$

(*ii*) Bandwidth $= f_2 - f_1 = 855 - 745 = \mathbf{110\ Hz.}$

Problems For Practice

1. A resistor of 50 Ω, an inductor of $20/\pi$ H and a capacitor of $5/\pi\ \mu$F are connected in series to a voltage supply of 230 V – 50 Hz. Find the impedance of the circuit. (Ans. 50 Ω)
2. A 40 Ω resistor, 3 mH inductor and 2 μF capacitor are connected in series to a 110 V, 5000 Hz a.c. source. Calculate the value of the current in the circuit. (Ans. 1.25 A)
3. A capacitor of unknown capacitance, a resistance of 100 Ω and an inductor of self-inductance $L(= 4/\pi^2)$ henry are connected in series across an a.c. source of 200 V and 50 Hz. Calculate the value of the capacitance and the current that flows in the circuit when the current is in phase with the voltage. **[CBSE OD 13C]** (Ans. 25 μF, 2 A)
4. A 50 μF capacitor, 0.05 H inductor and a 48 Ω resistor are connected in series with an a.c. source of emf, $\mathcal{E} = 310 \sin 314t$. Calculate the reactance of the circuit and tell its nature. What is the phase angle between the current and the applied emf ? [Ans. 48 Ω (capacitive), 45° (current leads voltage)]
5. An *LCR*-series circuit with $L = 100$ mH, $C = 100\ \mu$F, $R = 120\,\Omega$ is connected to an a.c. source of emf $\mathcal{E} = 30 \sin 100t$ volt. Find the impedance, peak current and resonant frequency of the circuit. (Ans. 150 Ω, 0.2 A, 50 Hz)
6. A 12 Ω resistance and an inductance of $0.05/\pi$ H with negligible resistance are connected in series. Across the ends of this is connected a 130 V alternating voltage of frequency 50 Hz. Calculate the alternating current in the circuit and the potential difference across the resistance and across the inductance. **[CBSE D 2000C]** (Ans. 10 A, 120 V, 50 V)
7. A capacitor, resistor of 5 Ω, and an inductor of 50 mH are in series with an a.c. source marked 100 V, 50 Hz. It is found that voltage is in phase with the current. Calculate the capacitance of the capacitor and the impedance of the circuit. **[CBSE D 99]** (Ans. 2.02×10^{-4} F, 5 Ω)
8. In the a.c. circuit shown in Fig. 7.38, the main supply has constant voltage but variable frequency. For what frequency will the voltage across the resistance *R* be maximum ? (Ans. 500 Hz)

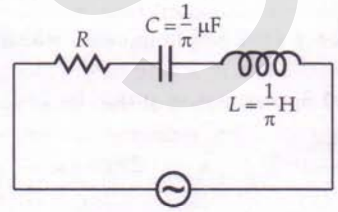


Fig. 7.38

9. An a.c. source of frequency 50 hertz is connected to a 50 mH inductor and a bulb. The bulb glows with some brightness. Calculate the capacitance of the capacitor to be connected in series with the circuit, so that the bulb glows with maximum brightness. **[CBSE D 2000]** (Ans. 2.02×10^{-4} F)
10. A 200 km long telegraph wire has a capacity of 0.014 μF per km. If it carries an alternating current of 50 kHz, what should be the value of an inductance required to be connected in series so that impedance is minimum ? Take $\pi = \sqrt{10}$. (Ans. 0.357 mH)
11. A series *LCR* circuit with $L = 4.0$ H, $C = 100\ \mu$F and $R = 60\,\Omega$ is connected to a variable frequency 240 V source as shown in Fig. 7.39.

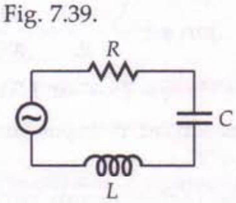


Fig. 7.39

Calculate :

(*i*) the angular frequency of the source which drives the circuit at resonance.

(*ii*) the current at the resonating frequency.

(*iii*) the rms potential drop across the inductor at resonance. **[CBSE D 12]**

[Ans. (*i*) 50 rad s^{-1} (*ii*) 4 A (*iii*) 800 V]

12. A series *LCR*-circuit is connected to an ac source (200 V, 50 Hz). The voltages across the resistor, capacitor and inductor are respectively 200 V, 250 V and 250 V.
   (*i*) The algebraic sum of the voltages across the three elements is greater than the voltage of the source. How is this paradox resolved ?
   (*ii*) Given the value of the resistance R is $40\,\Omega$, calculate the current in the circuit. **[CBSE F13]**
13. Figure 7.39 shows a series *LCR* circuit connected to a variable frequency 220 V source with $L = 80\,\text{mH}$, $C = 50\,\mu\text{F}$ and $R = 60\,\Omega$. Determine :
   (*i*) the source frequency which derives the circuit in resonance ;
   (*ii*) the quality factor (Q) of the circuit.
   [CBSE OD 14C] [Ans. (*i*) 80 Hz (*ii*) 0.67]
14. Compute the resonant frequency and the Q-factor of a series *LCR*-circuit having $L = 4.0$ H, $C = 36\,\mu\text{F} = 36 \times 10^{-6}$ F and $R = 10/3\,\Omega$. How can the sharpness of the resonance of the circuit be improved by a factor of 2 by reducing its full width at half maximum ? (Ans. 250/3 rad s^{-1}, to double Q, R should be halved)

HINTS

1. $X_L = 2\pi f L = 2\pi \times 50 \times \dfrac{20}{\pi} = 2000\,\Omega$

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 50 \times (5/\pi) \times 10^{-6}} = 2000\,\Omega$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{50^2 + (2000 - 2000)^2}$$

$= \mathbf{50\,\Omega}$.

2. Proceed as in Example 40 on page 7.25.
3. As current and voltage are in phase,

$$X_L = X_C \quad \text{or} \quad 2\pi f L = \frac{1}{2\pi f C}$$

$$\therefore \quad C = \frac{1}{4\pi^2 f^2 L} = \frac{1}{4\pi^2 (50)^2 (4/\pi^2)} = \frac{1}{40000}\,\text{F} = 25\,\mu\text{F}$$

$$I_{rms} = \frac{\mathcal{E}_{rms}}{Z} = \frac{\mathcal{E}_{rms}}{R} = \frac{200}{100} = \mathbf{2\,A}.$$

4. Proceed as in Exercise 7.21 on page 7.72.
5. Here $\mathcal{E}_0 = 30$ V, $\omega = 100$ rad s^{-1}

Reactance, $X = \dfrac{1}{\omega C} - \omega L$

$$= \frac{1}{100 \times 100 \times 10^{-6}} - 100 \times 100 \times 10^{-3}$$

$= 100 - 10 = 90\,\Omega$

$\therefore$ Impedance,

$$Z = \sqrt{R^2 + X^2} = \sqrt{20^2 + 90^2} = \mathbf{150\,\Omega}$$

Peak current, $I_0 = \dfrac{\mathcal{E}_0}{Z} = \dfrac{30}{150} = \mathbf{0.2\,A}$

Resonant frequency,

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

$$= \frac{1}{2\pi}\sqrt{\frac{1}{100 \times 10^{-3} \times 100 \times 10^{-6}}} = \mathbf{50\,Hz}$$

6. $Z = \sqrt{R^2 + (2\pi f L)^2}$

$$= \sqrt{12^2 + \left(2\pi \times 50 \times \frac{0.05}{\pi}\right)^2} = 13\,\Omega$$

$$\therefore \quad I_{rms} = \frac{\mathcal{E}_{rms}}{Z} = \frac{130}{13} = \mathbf{10\,A}$$

$$V_R = I_{rms} R = 10 \times 12 = \mathbf{120\,V}$$

$$X_L = 2\pi f L = 2\pi \times 50 \times \frac{0.05}{\pi} = 5\,\Omega$$

$$V_L = I_{rms} \cdot X_L = 10 \times 5 = \mathbf{50\,V}.$$

7. For current to be in phase with voltage,

$$X_C = X_L$$

$$\text{or} \quad \frac{1}{2\pi f C} = 2\pi f L$$

$$\therefore \quad C = \frac{1}{4\pi^2 f^2 L} = \frac{1 \times 7 \times 7}{4 \times 22 \times 22 \times (50)^2 \times 50 \times 10^{-3}}$$

$= \mathbf{2.02 \times 10^{-4}\,F}$.

Impedance,

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = R = \mathbf{5\,\Omega}.$$

8. For voltage across R to be maximum,

$$X_L = X_C \quad \text{or} \quad \frac{1}{2\pi f C} = 2\pi f L$$

$$\therefore \quad f = \frac{1}{2\pi\sqrt{LC}}$$

$$= \frac{1}{2\pi\sqrt{\frac{1}{\pi} \times \frac{1}{\pi} \times 10^{-6}}} = \mathbf{500\,Hz}.$$

9. Here $f = 50$ Hz, $L = 50$ mH $= 0.05$ H

The bulb will glow with maximum brightness when impedance is minimum *i.e.*,

$$X_C = X_L \quad \text{or} \quad \frac{1}{2\pi f C} = 2\pi f L$$

$$\therefore \quad C = \frac{1}{4\pi^2 f^2 L} = \frac{1}{4 \times 9.87 \times (50)^2 \times 0.05}$$

$= \mathbf{2.02 \times 10^{-4}\,F}$.

10. $C = 0.014 \times 200 = 2.8\,\mu\text{F} = 2.8 \times 10^{-6}$ F,
$f = 5$ kHz $= 5 \times 10^3$ Hz
Z is minimum when $X_L = X_C$

or $2\pi f L = \frac{1}{2\pi f C}$

$\therefore \quad L = \frac{1}{4\pi^2 f^2 C} = \frac{1}{4 \times 10 \times (5 \times 10^3)^2 \times 2.8 \times 10^{-6}}$

$= 0.357 \times 10^{-3}\,\text{H} = \mathbf{0.357\ mH.}$

11. Proceed as in Example 42 on page 7.25.

12. (*i*) The algebraic sum of the voltages across the three elements is greater than the voltage of the source because these voltages are not in same phase.

By using a phase diagram, we see that

$$V_{eff} = \sqrt{V_R^2 + (V_L - V_C)^2}$$

$$= \sqrt{200^2 + (250-250)^2} = 200\text{ V}$$

(*ii*) $I_{eff} = \frac{V_{eff}}{Z} = \frac{V_{eff}}{R} = \frac{200}{40} = \mathbf{5\,\Omega}.$

13. (*i*) $\omega_r = \frac{1}{\sqrt{LC}}$

$$= \frac{1}{\sqrt{80 \times 10^{-3} \times 50 \times 10^{-6}}} = 500\text{ rad s}^{-1}$$

$\therefore \quad f_r = \frac{\omega_r}{2\pi} = \frac{500}{2\pi} \simeq \mathbf{80\ Hz.}$

(*ii*) $Q = \frac{\omega_r L}{R}$

$$= \frac{500 \times 80 \times 10^{-3}}{60} = \frac{4}{6} = \frac{2}{3} \simeq \mathbf{0.67}.$$

14. Proceed as in Exercise 7.21 on page 7.72.

7.14 CHOKE COIL*

19. *What is a choke coil ? Explain its action in a.c. circuits. Why is it preferred to resistance in a.c. circuits ?*

Choke coil. *A choke coil is simply an inductor with large inductance which is used to reduce current in a.c. circuits without much loss of energy.*

Principle. The working of a choke is based on the fact that when a.c. flows through an inductor, current lags behind the emf by a phase angle of $\pi/2$ rad.

Construction. It is made of thick insulated copper wire wound closely in a large number of turns over a soft-iron laminated core. Choke coil offers a large reactance $X_L = 2\pi f L$ to the flow of a.c. and hence current is reduced. Laminated core reduces losses due to eddy currents.

Working. As shown in Fig. 7.40, a choke is put in series across an electrical appliance of resistance R and is connected to an a.c. source. This forms an LR-circuit.

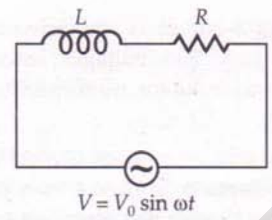


Fig. 7.40 Choke coil.

Average power dissipated per cycle in the circuit is

$$P_{av} = V_{eff}\, I_{eff} \cos\phi = V_{eff}\, I_{eff} \cdot \frac{R}{\sqrt{R^2 + \omega^2 L^2}}.$$

Inductance L of the choke coil is very large so that $R << \omega L$. Then

Power factor, $\cos\phi \simeq \frac{R}{\omega L} \simeq 0$

i.e., Average power dissipated by the coil is very small. As $Z = \sqrt{R^2 + \omega^2 L^2}$ is large, so current is reduced without appreciable wastage of power.

Preference of choke coil over the ohmic resistance. A choke coil reduces current in a.c. circuit without consuming any power. When an ohmic resistance is used, current reduces but energy losses occur due to heating. So a choke coil is preferred.

Uses. The most common use of choke coil is in the fluorescent tubes with a.c. mains. If the tube is connected directly across 220 V source, it would draw large currents which would damage the tube. With the use of choke coil, the voltage is reduced to an appropriate value, without wasting any power. Choke coils are also used in various electronic circuits, mercury lamp and in sodium vapour lamp.

7.15 POWER IN AN A.C. CIRCUIT

20. *Define power for an a.c. circuit. Derive an expression for the average power of a series LCR-circuit connected to an a.c. source. Discuss the various special cases.*

Power in an a.c. circuit. *The rate at which electric energy is consumed in an electric circuit is called its power.* In a d.c. circuit, power is given by the product of voltage and current. But in an a.c. circuit, both voltage $\mathcal{E}$ and current I vary sinusoidally with time and are generally not in phase. So for an a.c. circuit, we define ***instantaneous power*** *as the product of the instantaneous voltage and instantaneous current.*

Suppose in an a.c. circuit, the voltage and current at any instant are given by

$$\mathcal{E} = \mathcal{E}_0 \sin\omega t \quad \text{and} \quad I = I_0 \sin(\omega t - \phi)$$

where ϕ is the phase angle by which the voltage $\mathcal{E}$ leads the current I.

The instantaneous power is given by

$$P = \mathcal{E}I = \mathcal{E}_0 I_0 \sin \omega t \,.\, \sin(\omega t - \phi)$$

$$= \frac{\mathcal{E}_0 I_0}{2} [2 \sin \omega t . \sin(\omega t - \phi)]$$

$$= \frac{\mathcal{E}_0 I_0}{2} [\cos \phi - \cos(2\omega t - \phi)]$$

$$[\because 2 \sin A \sin B = \cos(A - B) - \cos(A + B)]$$

Average power dissipated per cycle

$$= \text{Average of } \frac{\mathcal{E}_0 I_0}{2} [\cos \phi - \cos(2\omega t - \phi)].$$

The second cosine term $[\cos(2\omega t - \phi)]$ is time-dependent. Its average over a cycle is zero.

$$\therefore \quad P_{av} = \frac{\mathcal{E}_0 I_0}{2} \cos \phi$$

or $$P_{av} = \frac{\mathcal{E}_0}{\sqrt{2}} \cdot \frac{I_0}{\sqrt{2}} \cdot \cos \phi$$

or $$P_{av} = \mathcal{E}_{rms} I_{rms} \cos \phi = \mathcal{E}_{rms} I_{rms} \cdot \frac{R}{Z}$$

Special Cases

1. Pure resistive circuit. Here the voltage and current are in same phase, *i.e.*, $\phi = 0$ and $\cos \phi = 1$.

$$\therefore \quad P_{av} = \mathcal{E}_{rms} \cdot I_{rms} \times 1 = \mathcal{E}_{rms} \cdot I_{rms} = \frac{\mathcal{E}_{rms}^2}{R}$$

2. Pure inductive circuit. Here the voltage leads the current in phase by $\frac{\pi}{2}$, *i.e.*, $\phi = \frac{\pi}{2}$.

$$\therefore \quad P_{av} = \mathcal{E}_{rms} \cdot I_{rms} \cos \frac{\pi}{2} = 0$$

Thus the average power consumed in an inductive circuit over a complete cycle is zero.

3. Pure capacitive circuit. Here the voltage lags behind the current in phase by $\frac{\pi}{2}$, *i.e.*, $\phi = -\frac{\pi}{2}$.

$$\therefore \quad P_{av} = \mathcal{E}_{rms} \cdot I_{rms} \cos\left(-\frac{\pi}{2}\right) = 0$$

Thus the average power consumed in a capacitive circuit over a complete cycle is also zero.

4. Series *LCR*-circuit. For a series *LCR*-circuit, $P_{av} = \mathcal{E}_{rms} I_{rms} \cos \phi$, where $\phi = \tan^{-1} \frac{X_L - X_C}{R}$. Sometimes, ϕ may have a non-zero value for series *LR*-, *LC*- and *LCR*-circuits. So power is consumed in such circuits, but only in the resistor *R*.

5. Power dissipated at resonance in *LCR*-circuit. At resonance, $X_L = X_C$, and $\phi = 0$. So $\cos \phi = 1$, and $P_{av} = \mathcal{E}_{rms} I_{rms}$. That is, *maximum power is dissipated in the circuit (through R) at resonance.*

7.16 POWER FACTOR

21. *What do you mean by power factor of an a.c. circuit ? Write an expression for it. When is the value of the power factor of an a.c. circuit minimum and maximum ?*

Power Factor. The average power of an a.c. circuit is given by

$$P_{av} = \mathcal{E}_{rms} \cdot I_{rms} \cos \phi$$

Average power

= Virtual emf × Virtual current × cos ϕ

The product $\mathcal{E}_{rms} \cdot I_{rms}$ does not give the actual power and is called *apparent power*. It gives *actual* or *true power* only when multiplied by factor cos ϕ. The factor cos ϕ is called the *power factor* of an *a.c.* circuit.

$\therefore$ True power = Apparent power × power factor

Thus power factor may be defined as the ratio of the true power to the apparent power of an a.c. circuit. Its value varies from 0 to 1. The power factor of a series *LCR*-circuit is given by

$$\cos \phi = \frac{R}{Z} = \frac{R}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}$$

For a purely inductive or capacitive circuit, $\phi = 90°$.

$\therefore$ Power factor = $\cos 90° = 0$.

Thus the power factor assumes the *minimum value* for a purely inductive or capacitive circuit.

For a purely resistive circuit, $\phi = 0°$.

$\therefore$ Power factor = $\cos 0° = 1$

Thus the power factor assumes the *maximum value* for a purely resistive circuit.

7.17 WATTLESS CURRENT

22. *What is wattless current ? When is the current in an a.c. circuit wattless ?*

Wattless current. *The current in a.c. circuit is said to be wattless if the average power consumed in the circuit is zero.* The average power of an a.c. circuit is given by

$$P_{av} = \mathcal{E}_{rms} I_{rms} \cos \phi$$

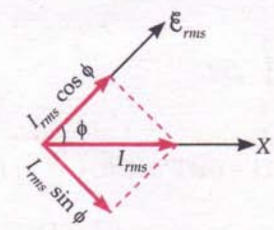


Fig. 7.41 A phasor diagram.

Figure 7.41 shows the phase angle ϕ between $\mathcal{E}_{rms}$ and I_{rms}. The current I_{rms} can be resolved into *two* components :

(*a*) *Component* $I_{rms}\cos\phi$ *along* $\mathcal{E}_{rms}$. As the phase angle between $I_{rms}\cos\phi$ and $\mathcal{E}_{rms}$ is zero, therefore

$$P_{av} = \mathcal{E}_{rms}\,(I_{rms}\cos\phi)\cos 0 = \mathcal{E}_{rms}\,I_{rms}\cos\phi$$

(*b*) *Component* $I_{rms}\sin\phi$ *normal to* $\mathcal{E}_{rms}$. As the phase angle between $I_{rms}\sin\phi$ and $\mathcal{E}_{rms}$ is $\frac{\pi}{2}$, therefore

$$P_{av} = \mathcal{E}_{rms}\,(I_{rms}\sin\phi).\cos\frac{\pi}{2} = 0$$

We call the component $I_{rms}\sin\phi$ as the ***idle*** or ***wattless current*** because it does not consume any power in a.c. circuit. This happens in a purely inductive or capacitive circuit in which the voltage and current differ by a phase angle of $\frac{\pi}{2}$, *i.e.*, $\phi = \pm\frac{\pi}{2}$, so that

$$P_{av} = \mathcal{E}_{rms}\,I_{rms}\cos(\pm\pi/2) = 0$$

Thus the current in the circuit has no power. It flows sometimes along the voltage and sometimes against the voltage, so that the net work done per cycle is zero. For example, when the secondary of a transformer is open, the current in the primary is almost wattless.

7.18 AVERAGE POWER ASSOCIATED WITH A RESISTOR

23. *Prove that an ideal resistor dissipates power of* V_{rms}^2/R *in an a.c. circuit.*

Average power associated with resistor. In case of a pure resistor, the voltage and current are always in same phase. So we can write the instantaneous values of voltage and current as follows :

$$V = V_0\sin\omega t \quad \text{and} \quad I = I_0\sin\omega t$$

Work done in small time dt will be

$$dW = P\,dt = VI\,dt = V_0 I_0\sin^2\omega t\,dt$$

$$= \frac{V_0 I_0}{2}(1-\cos 2\omega t)\,dt$$

The average power dissipated per cycle in the resistor will be

$$P_{av} = \frac{W}{T} = \frac{1}{T}\int_0^T dW$$

$$= \frac{V_0 I_0}{2T}\int_0^T (1-\cos 2\omega t)\,dt = \frac{V_0 I_0}{2T}\left[t - \frac{\sin 2\omega t}{2\omega}\right]_0^T$$

$$= \frac{V_0 I_0}{2T}[(T-0)-0] = \frac{V_0 I_0}{2} = \frac{V_0^2}{2R}$$

or $$P_{av} = \frac{V_0 I_0}{\sqrt{2}\,\sqrt{2}} = V_{rms}\,I_{rms} = \frac{V_{rms}^2}{R} \qquad \left[\because \frac{V_0}{\sqrt{2}} = V_{rms}\right]$$

7.19 ENERGY AND AVERAGE POWER ASSOCIATED WITH A PURE INDUCTOR

24. *Derive an expression for the energy stored in an inductor.*

Energy stored in an inductor. When an inductor is connected to a source of emf, the current starts growing through it. An induced emf is set up in the inductor which opposes the growth of current through it. The external source has to expend energy in building up the current through the inductor against the induced emf. This energy is stored in the inductor as magnetic field energy.

Let I be the current through the inductor L at any instant t. The current rises at the rate dI/dt. So the induced emf is

$$\mathcal{E} = -L\frac{dI}{dt}$$

The work done against the induced emf in small time dt is

$$dW = P\,dt = -\mathcal{E}I\,dt = +L\frac{dI}{dt}.\,I\,dt = LI\,dI$$

The total work done in building up the current from 0 to I_0 is

$$W = \int dW = \int_0^{I_0} LI\,dI = L\left[\frac{I^2}{2}\right]_0^{I_0} = \frac{1}{2}LI_0^2$$

This work done is stored as the magnetic field energy U in the inductor.

$$\therefore \quad U = \frac{1}{2}LI_0^2$$

25. *Prove that an ideal inductor connected to an a.c. source does not dissipate any power.*

Average power associated with an inductor. When a.c. is applied to an ideal inductor, current lags behind the voltage in phase by $\pi/2$ radian. So we can write the instantaneous values of voltage and current as follows :

$$V = V_0\sin\omega t$$

and $$I = I_0\sin\left(\omega t - \frac{\pi}{2}\right)$$

$$= -I_0\sin\left(\frac{\pi}{2} - \omega t\right) = -I_0\cos\omega t$$

Work done in small time dt is

$$dW = P\,dt = -V_0 I_0\sin\omega t\cos\omega t\,dt$$

$$= -\frac{V_0 I_0}{2}\sin 2\omega t\,dt$$

The average power dissipated per cycle in the inductor is

$$P_{av} = \frac{W}{T} = \frac{1}{T}\int_0^T dW$$

$$= -\frac{V_0 I_0}{2T}\int_0^T \sin 2\omega t \, dt$$

$$= +\frac{V_0 I_0}{2T}\left[\frac{\cos 2\omega t}{2\omega}\right]_0^T = \frac{V_0 I_0}{4T\omega}\left[\cos \frac{4\pi}{T} t\right]_0^T$$

$$= \frac{V_0 I_0}{4T\omega}[\cos 4\pi - \cos 0] = \frac{V_0 I_0}{4T\omega}[1-1]$$

$$= \mathbf{0}$$

Thus *the average power dissipated per cycle in an inductor is zero.*

For Your Knowledge

- The energy stored in an inductor resides in the region of its magnetic field.
- The average power consumed per cycle in an inductor connected to an a.c. source is zero. The physical meaning of this result is as follows. During the first quarter of each current cycle, as the current increases, the magnetic flux through the inductor builds up and energy is stored in the inductor from the external source. In the next quarter of cycle, as the current decreases, the flux decreases and the stored energy is returned to the source. Thus, in half cycle, no net power is consumed by the inductor.

7.20 ENERGY AND AVERAGE POWER ASSOCIATED WITH A PURE CAPACITOR

26. *Derive an expression for the energy stored in a capacitor.*

Energy stored in a capacitor. Consider a capacitor of capacitance C. Suppose the displacement of charge q from one plate to another sets up a potential difference V between its plates. Then

$$V = \frac{q}{C}$$

Suppose now a small additional charge dq be displaced from one plate to another. Then work done is

$$dW = V\,dq = \frac{q}{C}\,dq$$

$\therefore$ Total work done in displacing a charge q from one plate to another is

$$W = \int_0^q dW = \int_0^q \frac{q}{C}\,dq = \frac{1}{2}\frac{q^2}{C}$$

This energy is stored as the electrostatic energy U in the capacitor.

$$\therefore \quad U = \frac{1}{2}\frac{q^2}{C} = \frac{1}{2}CV^2 \qquad [\because q = CV]$$

27. *Prove that an ideal capacitor connected to an a.c. source does not dissipate any power.*

Average power associated with a capacitor. When an a.c. is applied to a capacitor, the current leads the voltage in phase by $\pi/2$ radian. So we write the expressions for instantaneous voltage and current as follows :

$$V = V_0 \sin \omega t$$

and

$$I = I_0 \sin\left(\omega t + \frac{\pi}{2}\right) = I_0 \cos \omega t$$

Work done in the circuit in small time dt will be

$$dW = P dt = VI\,dt = V_0 I_0 \sin \omega t \cos \omega t \, dt$$

$$= \frac{V_0 I_0}{2}\sin 2\omega t \, dt$$

The average power dissipated per cycle in the capacitor is

$$P_{av} = \frac{W}{T} = \frac{1}{T}\int_0^T dW = \frac{V_0 I_0}{2T}\int_0^T \sin 2\omega t \, dt$$

$$= \frac{V_0 I_0}{2T}\left[-\frac{\cos 2\omega t}{2\omega}\right]_0^T$$

$$= -\frac{V_0 I_0}{4T\omega}\left[\cos \frac{4\pi}{T} t\right]_0^T$$

$$= -\frac{V_0 I_0}{4T\omega}[\cos 4\pi - \cos 0]$$

$$= -\frac{V_0 I_0}{4T\omega}[1-1] = \mathbf{0}$$

Thus *the average power dissipated per cycle in a capacitor is zero.*

For Your Knowledge

- Energy stored in a capacitor resides in the region of its electric field.
- The external source has to supply an energy $\frac{1}{2}CV^2$ to charge a capacitor to a p.d. V but this energy is returned back during the discharging process. When the capacitor is connected across an a.c. source, it absorbs energy from the source for a quarter cycle as it is charged. It returns energy to source in the next quarter cycle as it is discharged. Thus in a half cycle, no net power is consumed by the capacitor.

Examples based on Energy and Power associated with A.C. Circuits

Formulae Used

1. Average power consumed per cycle in any a.c circuit, $P_{av} = \mathcal{E}_{rms}\, I_{rms} \cos\phi$,

   $\mathcal{E}_{rms} \cdot I_{rms}$ is the apparent power

2. Power factor, $\cos\phi = \dfrac{R}{Z} = \dfrac{R}{\sqrt{R^2 + (X_L - X_C)^2}}$

3. Average power consumed per cycle in a pure resistive circuit,

   $$P_{av} = \frac{\mathcal{E}_0^2}{2R} = \mathcal{E}_{rms} \cdot I_{rms} = \frac{\mathcal{E}_{rms}^2}{R}$$

4. Energy stored in an inductor, $U = \dfrac{1}{2} LI^2$

5. Average power consumed per cycle in pure inductive circuit = 0.

6. Energy stored in a capacitor, $U = \dfrac{1}{2} CV^2 = \dfrac{1}{2}\dfrac{Q^2}{C}$

7. Average power consumed per cycle in a pure capacitive circuit = 0.

8. For an LCR-circuit in resonance,

   $$X_L = X_C \text{ and } f_r = \frac{1}{2\pi\sqrt{LC}}.$$

Units Used

Power P_{av} is in watt, current I_{rms} in ampere, voltage $\mathcal{E}_{rms}$ in volt, inductance L in henry, capacitance C in farad, energy U in joule and R, X_L, X_C and Z are all in ohm.

Example 50. *A light bulb is rated at 100 W for a 220 V supply. Find (a) the resistance of the bulb ; (b) the peak voltage of the source ; and (c) the rms current through the bulb.* [NCERT]

Solution. Here, $P_{av} = 100$ W, $V_{rms} = 220$ V

(a) $R = \dfrac{V_{rms}^2}{P_{av}} = \dfrac{(220)^2}{100} = \mathbf{484\ \Omega}$.

(b) $V_0 = \sqrt{2}\, V_{rms} = 1.414 \times 220 = \mathbf{311\ V}$.

(c) $I_{rms} = \dfrac{P_{av}}{V_{rms}} = \dfrac{100}{220} = \mathbf{0.45\ A}$.

Example 51. *A capacitor and a resistor are connected in series with an a.c. source. If the potential differences across C, R are 120 V, 90 V respectively and if the rms current of the circuit is 3 A, calculate the (i) impedance, (ii) power factor of the circuit.* [CBSE D 06C]

Solution. $\mathcal{E}_{rms} = \sqrt{V_R^2 + V_C^2} = \sqrt{90^2 + 120^2}$

$= \sqrt{22500} = 150$ V

$I_{rms} = 3$ A

(i) Impedance, $Z = \dfrac{\mathcal{E}_{rms}}{I_{rms}} = \dfrac{150}{3} = \mathbf{50\ \Omega}$.

(ii) Power factor, $\cos\phi = \dfrac{V_R}{\mathcal{E}_{rms}} = \dfrac{90}{150} = \mathbf{0.6}$.

Example 52. *In the following circuit, calculate, (i) the capacitance 'C' of the capacitor, if the power factor of the circuit is unity, and (ii) also calculate the Q-factor of the circuit.* [CBSE D 06C]

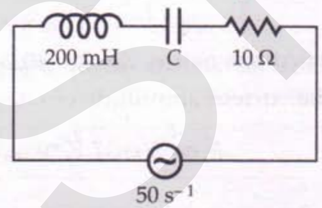


Fig. 7.42

Solution. (i) Power factor, $\cos\phi = \dfrac{R}{Z} = 1$

or $Z = R$

$\therefore$ $X_C = X_L$ or $\dfrac{1}{2\pi fC} = 2\pi fL$

or $C = \dfrac{1}{4\pi^2 f^2 L} = \dfrac{1}{4 \times 9.87 \times (50)^2 \times 200 \times 10^{-3}}$

$= 5 \times 10^{-5}\,\text{F} = \mathbf{50\ \mu F}$.

(ii) Q-factor $= \dfrac{1}{R}\sqrt{\dfrac{L}{C}} = \dfrac{1}{10}\sqrt{\dfrac{200 \times 10^{-3}}{5 \times 10^{-5}}} = \mathbf{6.32}$.

Example 53. *An alternating voltage $\mathcal{E} = 200 \sin 300\, t$ is applied across a series combination of $R = 10\,\Omega$ and an inductor of 800 mH. Calculate :*

(i) impedance of the circuit
(ii) peak value of current in the circuit
(iii) power factor of the circuit. [CBSE OD 94]

Solution. Given $\mathcal{E} = 200 \sin 300\, t$

Comparing with equation, $\mathcal{E} = \mathcal{E}_0 \sin \omega t$, we find that

$\mathcal{E}_0 = 200$ V, $\omega = 300$ rad s^{-1}

(i) Impedance,

$Z = \sqrt{R^2 + \omega^2 L^2} = \sqrt{10^2 + (300)^2 \times (800 \times 10^{-3})^2}$

$= \mathbf{240.2\ \Omega}$.

(ii) Peak value of current,

$I_0 = \dfrac{\mathcal{E}_0}{Z} = \dfrac{200}{240.2} = \mathbf{0.832\ A}$.

(iii) Power factor, $\cos\phi = \dfrac{R}{Z} = \dfrac{10}{240.2} = \mathbf{0.041}$.

Example 54. *A 200 V variable frequency a.c. source is connected to a series combination of $L = 5$ H, $C = 80\ \mu F$ and $R = 40\ \Omega$. Calculate (i) angular frequency of the source to get maximum current in the circuit, (ii) the current amplitude at resonance and (iii) the power dissipated in the circuit.* [CBSE OD 02]

Solution. Here $\mathcal{E}_{rms} = 200$ V, $L = 5$ H, $C = 80\ \mu$F, $R = 40\ \Omega$

(*i*) Resonant angular frequency,

$$\omega_r = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{5 \times 80 \times 10^{-6}}} = \mathbf{50\ rad\ s^{-1}}.$$

(*ii*) At resonance, $Z = R = 40\ \Omega$

The current amplitude at resonance,

$$I_0 = \frac{\mathcal{E}_0}{R} = \frac{\sqrt{2}\,\mathcal{E}_{rms}}{R} = \frac{1.414 \times 200}{40} = \mathbf{7.07\ A}.$$

(*iii*) Power dissipated in the circuit

$$= \frac{\mathcal{E}_{rms}^2}{R} = \frac{(200)^2}{40} = \mathbf{1000\ W}.$$

Example 55. *A sinusoidal voltage of peak value 283 V and frequency 50 Hz is applied to a series LCR circuit in which $R = 3\ \Omega$, $L = 25.48$ mH, and $C = 796\ \mu F$. Find (a) the impedance of the circuit, (b) the phase difference between the voltage across the source and the currents, (c) the power dissipated in the circuit, and (d) the power factor.* [NCERT]

Solution. Here $\mathcal{E}_0 = 283$ V, $f = 50$ Hz, $R = 3\ \Omega$, $L = 25.48 \times 10^{-3}$ H, $C = 796 \times 10^{-6}$ F

(*a*) $X_L = 2\pi f L = 2 \times 3.14 \times 50 \times 25.48 \times 10^{-3} = 8\ \Omega$.

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2 \times 3.14 \times 50 \times 796 \times 10^{-6}} = 4\ \Omega$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{3^2 + (8-4)^2} = \mathbf{5\ \Omega}.$$

(*b*) Phase difference ϕ is given by

$$\tan\phi = \frac{X_L - X_C}{R} = \frac{8-4}{3} = \frac{4}{3}$$

$$\therefore \quad \phi = \tan^{-1}\frac{4}{3} = \mathbf{53.1^\circ}.$$

Thus the current in the circuit lags behind the voltage across the source by a phase angle of 53.1°.

(*c*) $I_{rms} = \dfrac{I_0}{\sqrt{2}} = \dfrac{\mathcal{E}_0}{\sqrt{2}\,R} = \dfrac{1}{1.414} \times \dfrac{283}{5} = 40$ A

Power dissipated in the circuit,

$$P_{av} = I_{rms}^2 R = (40)^2 \times 3 = \mathbf{4800\ W}.$$

(*d*) Power factor $= \cos\phi = \cos 53.1^\circ = \mathbf{0.6}$.

Example 56. *Suppose the frequency of the source in the previous example can be varied. (a) What is the frequency of the source at which resonance occurs ? (b) Calculate the impedance, the current, and the power dissipated at the resonant condition.* [NCERT]

Solution. (*a*) Resonant frequency of the source,

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

$$= \frac{1}{2 \times 3.14\sqrt{25.48 \times 10^{-3} \times 796 \times 10^{-6}}}$$

$$= \frac{221.1}{2 \times 3.14} = \mathbf{35.4\ Hz}.$$

(*b*) At resonance, the impedance is

$$Z = R = 3\ \Omega$$

The rms current at resonance,

$$I_{rms} = \frac{\mathcal{E}_{rms}}{Z} = \frac{0.707\,\mathcal{E}_0}{R} = \frac{0.707 \times 283}{3}$$

$$= \mathbf{66.7\ A}$$

The power dissipated at resonance is

$$P_{av} = I_{rms}^2 R = (66.7)^2 \times 3\ \text{W} = \mathbf{13.35\ kW}$$

Obviously, the power dissipated at resonance is more than the power dissipated in the non-resonant condition of the above example.

Example 57. *A virtual current of 4 A flows in a coil when it is connected in a circuit having alternating current of frequency 50 Hz. Power consumed in the coil is 240 W. Calculate the inductance of the coil if the virtual potential difference across it is 100 V.*

Solution. Here $I_{eff} = 4$ A, $f = 50$ Hz, $V_{eff} = 100$ V, $P = 240$ W

$$P = I_{eff}^2\,R \quad \therefore \quad 240 = 16\,R$$

or $$R = \frac{240}{16} = 15\ \Omega\ ;\ Z = \frac{V_{eff}}{I_{eff}} = \frac{100}{4} = 25\ \Omega$$

But $$Z = \sqrt{R^2 + \omega^2 L^2} \quad \text{or} \quad Z^2 = R^2 + \omega^2 L^2$$

$$\therefore \quad L = \frac{\sqrt{Z^2 - R^2}}{\omega} = \frac{\sqrt{25^2 - 15^2}}{2\pi \times 50}$$

$$= \frac{20}{100\pi} = \mathbf{\frac{1}{5\pi}\ H}. \qquad [\because \omega = 2\pi f]$$

Example 58. *A circuit draws a power of 550 W from a source of 220 V, 50 Hz. The power factor of the circuit is 0.8. The current in the circuit lags behind the voltage. Show that a capacitor of about $\frac{1}{42\pi} \times 10^{-2}$ F will have to be connected to bring its power factor to unity.*

Solution. As $P_{av} = V_{eff} \cdot I_{eff} \cos\phi$

$$\therefore \quad I_{eff} = \frac{P_{av}}{V_{eff}\cos\phi} = \frac{550}{220 \times 0.8} = \frac{25}{8}\ \text{A}$$

$$R = \frac{P_{av}}{I_{eff}^2} = \frac{550 \times 8 \times 8}{25 \times 25} = \frac{22 \times 64}{25}\,\Omega \qquad [\because P_{av} = I_{eff}^2 R]$$

$$\tan\phi = \frac{\sin\phi}{\cos\phi} = \frac{\sqrt{1-(0.8)^2}}{0.8} = \frac{0.6}{0.8} = \frac{3}{4}$$

But $\tan\phi = \dfrac{X_L}{R}$

$$\therefore \quad X_L = \tan\phi \,.\, R = \frac{3}{4} \times \frac{22 \times 64}{25} \simeq 42\,\Omega$$

For power factor to be unity,

$$X_L = X_C \quad \text{or} \quad \omega L = \frac{1}{\omega C}$$

or $$C = \frac{1}{\omega^2 L} = \frac{1}{\omega X_L} = \frac{1}{2\pi f} \cdot \frac{1}{X_L} = \frac{1}{100\,\pi} \cdot \frac{1}{42} \qquad [\because \omega L = X_L]$$

or $$C = \frac{\mathbf{1}}{\mathbf{42\,\pi}} \times \mathbf{10^{-2}\ F.}$$

Example 59. *An emf $\mathcal{E} = 100 \sin 314\, t$ is applied across a pure capacitor of 637 μF. Find (i) the instantaneous current I (ii) instantaneous power P (iii) the frequency of power and (iv) the maximum energy stored in the capacitor.*

Solution. (*i*) Given $\mathcal{E} = 100 \sin 314\, t$ volt

As the current in a capacitor leads the voltage by 90°, so the instantaneous current is given by

$$I = I_0 \sin(314\,t + 90°) = I_0 \cos 314\, t$$

where $$I_0 = \frac{\mathcal{E}_0}{X_C} = \frac{\mathcal{E}_0}{1/\omega C} = \mathcal{E}_0\, \omega C$$

But $\mathcal{E}_0 = 100$ V, $\omega = 314$ rad s^{-1}, $C = 637 \times 10^{-6}$ F

$$\therefore \quad I_0 = 100 \times 314 \times 637 \times 10^{-6} = 20\ \text{A}$$

Hence **$I = 20 \cos 314\, t$ ampere.**

(*ii*) Instantaneous power,

$$P = \mathcal{E} I = 100 \sin 314\,t \times 20 \cos 314\,t = \mathbf{1000 \sin 628\, t\ watt.}$$

(*iii*) Angular frequency of power, $\omega_p = 628$ rad s^{-1}

$\therefore$ Frequency of power, $f_p = \dfrac{\omega_p}{2\pi} = \dfrac{628}{2\pi} = \mathbf{100\ Hz.}$

(*iv*) The maximum energy stored in the capacitor is

$$U_0 = \frac{1}{2} C \mathcal{E}_0^2 = \frac{1}{2} \times 637 \times 10^{-6} \times (100)^2 = \mathbf{3.185\ J.}$$

Example 61. *A series LCR circuit is made by taking $R = 100\,\Omega$, $L = \dfrac{2}{5}$ H, $C = \dfrac{100}{\pi}$ μF. This series combination is connected across an a.c. source of 220 V, 50 Hz. Calculate (i) the impedance of the circuit and (ii) the peak value of current flowing in the circuit. Calculate the power factor of this circuit and compare this value with the one at its resonant frequency.* [CBSE OD 08C]

Solution. Here $R = 100\,\Omega$, $L = \dfrac{2}{\pi}$ H, $C = \dfrac{100}{\pi}$ μF, $\mathcal{E}_{rms} = 220$ V, $f = 50$ Hz

$$\therefore \quad X_L = 2\pi f L = 2\pi \times 50 \times \frac{2}{\pi} = 200\,\Omega$$

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 50 \times \dfrac{100}{\pi} \times 10^{-6}} = 100\,\Omega$$

(*i*) Impedance,

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{100^2 + (200-100)^2} = 100\sqrt{2} = 100 \times 1.414 = \mathbf{141.4\,\Omega.}$$

(*ii*) Peak value of current,

$$I_0 = \frac{\mathcal{E}_0}{Z} = \frac{\sqrt{2}\,\mathcal{E}_{rms}}{Z} = \frac{\sqrt{2} \times 220}{100\sqrt{2}} = \mathbf{2.2\ A}$$

Power factor of the given circuit

$$\cos\phi = \frac{R}{Z} = \frac{100}{100\sqrt{2}} = \frac{\mathbf{1}}{\mathbf{\sqrt{2}}}$$

At resonant frequency, $Z = R$, so the power factor will be $\cos\phi = \dfrac{R}{Z} = \dfrac{R}{R} = \mathbf{1.}$

Example 62. *The current in a coil of self-inductance 2.0 H is increasing according to $I = 2 \sin t^2$ ampere. Find the amount of energy spent during the period when the current changes from zero to 2 A.* [Roorkee 91]

Solution. The energy stored in the coil when the current changes from 0 to 2 A is

$$U = \frac{1}{2} L I_0^2 = \frac{1}{2} \times 2 \times (2)^2 = \mathbf{4\ J.}$$

Example 63. *A 100 μF capacitor is charged with a 50 V source supply. Then source supply is removed and the capacitor is connected across an inductance, as a result of which 5 A current flows through the inductance. Calculate the value of the inductance.*

Solution. Energy stored in the capacitor = Energy stored in the inductor

or $$\frac{1}{2} C V^2 = \frac{1}{2} L I^2$$

$$\therefore \quad L = \frac{CV^2}{I^2} = \frac{100 \times 10^{-6} \times (50)^2}{5^2} = \mathbf{0.01\ H.}$$

Example 64. *A coil has an inductance of 0.7 H and is joined in series with a resistance of 220 Ω. Find the wattless component of the current in the circuit, when an alternating emf of 220 V at a frequency of 50 Hz is supplied to it.*

Solution. Here $L = 0.7$ H, $R = 220\,\Omega$, $\mathcal{E}_{rms} = 220$ V, $f = 50$ Hz, $X_L = 2\pi fL = 2\pi \times 50 \times 0.7 = 220\,\Omega$

$$I_{rms} = \frac{\mathcal{E}_{rms}}{\sqrt{R^2 + X_L^2}} = \frac{220}{\sqrt{220^2 + 220^2}} = \frac{1}{\sqrt{2}}\text{ A}$$

Now $\tan\phi = \frac{X_L}{R} = \frac{220}{220} = 1 \quad \therefore \quad \phi = 45°$

Wattless component of current $= I_{rms} \sin\phi$

$$= \frac{1}{\sqrt{2}} \times \sin 45° = \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} = \frac{1}{2} = \mathbf{0.5\ A.}$$

Problems For Practice

1. An ammeter shows that an alternator is delivering 20 A. The voltmeter reads 220 V, while a wattmeter shows that 4 kW of power is being delivered. Find the power factor. (Ans. 0.91)
2. A circuit containing an 80 mH inductor and a 250 μF capacitor in series is connected to a 240 V, 100 rad/s supply. The resistance of the circuit is negligible.
   (*i*) Obtain rms value of current.
   (*ii*) What is the total average power consumed by the circuit ? **[CBSE D 15C]**
   [Ans. (*i*) 7.5 A (*ii*) 0]
3. An a.c. circuit has a resistance and an inductance connected in series as shown in Fig. 7.43. Calculate the current and the power factor in the circuit. (Ans. 0.1325 A, 0.053)

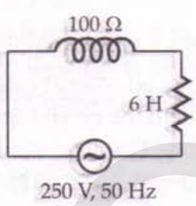


Fig. 7.43

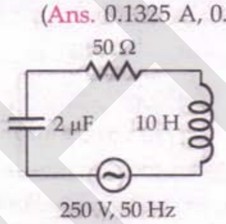


Fig. 7.44

4. Calculate the current and power factor of the circuit shown in Fig. 7.44. (Ans. 0.16 A, 0.032)
5. In the circuit shown in Fig. 7.45, the potential differences across resistance, capacitance and inductance are given. Find the emf of the source of alternating current and power factor of the circuit. (Ans. 100 V, 0.8)

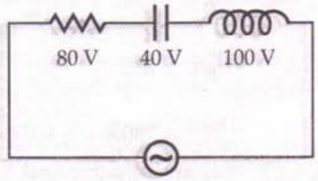


Fig. 7.45

6. A group of electric bulbs has a power rating of 300 W. An a.c. voltage, $V = 141.4 \sin(314t + \pi/3)$ is applied to the group. Calculate the effective current. (Ans. 3 A)
7. An alternating voltage and the corresponding current in a circuit are given by
   $\mathcal{E} = 110 \sin(\omega t + \pi/6)$ and $I = 5 \sin(\omega t - \pi/6)$
   respectively. Find the impedance and the average power dissipation in it. (Ans. 22 Ω, 137.5 W)
8. An inductor 200 mH, capacitor 500 μF, resistor 10 Ω are connected in series with a 100 V, variable frequency a.c. source. Calculate the **[CBSE D 08]**
   (*i*) frequency at which the power factor of the circuit is unity,
   (*ii*) current amplitude at this frequency,
   (*iii*) *Q*-factor. (Ans. $\frac{50}{\pi}$ Hz, 14.14 A, 2)
9. An inductor of unknown value, a capacitor of 100 μF and a resistor of 10 Ω are connected in series to a 200 V, 50 Hz a.c. source. It is found that the power factor of the circuit is unity. Calculate the inductance of the inductor and the current amplitude. **[CBSE D 08]** (Ans. 101.3 mH, 28.28 A)
10. Find the value of the phase lag/lead between the current and voltage in the given series *LCR* circuit. Without making any other change, find the value of the additional capacitor, such that when 'suitably joined' to the capacitor ($C = 2\mu$F) as shown in Fig. 7.46, would make the power factor of this circuit unity. **[CBSE OD 15]**

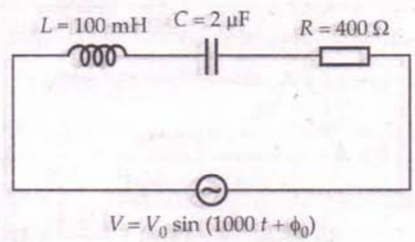


Fig. 7.46

11. A circuit containing a 80 mH inductor and a 60 μF capacitor in series is connected to a 230 V, 50 Hz supply. If the circuit has a resistance of 15 Ω, obtain the average power transferred to each element of the circuit and the total power absorbed.
    [Haryana 01] (Ans. $P_{av}^R = 790.6$ W, $P_{av}^L = 0$, $P_{av}^C = 0$, $P_{av}^{total} = 790.6$ W)
12. A resistor R and an element X are connected in series to an ac source of voltage. The voltage is found to lead the current in phase by $\pi/4$. If X is replaced by another element Y, the voltage lags behind the current by $\pi/4$.
    (*i*) Identify elements X and Y.
    (*ii*) When both X and Y are connected in series with R to the same source, will the power dissipated in the circuit be maximum or minimum ? Justify your answer. **[CBSE F 13]**

HINTS

2. (i) $X_L = \omega L = 100 \times 80 \times 10^{-3} = 8\,\Omega$

$$X_C = \frac{1}{\omega C} = \frac{1}{100 \times 250 \times 10^{-6}} = 40\,\Omega$$

Impedance, $Z = X_C - X_L = 32\,\Omega$

$$I_{rms} = \frac{\mathcal{E}_{eff}}{Z} = \frac{240}{32}\text{ A} = \mathbf{7.5\ A.}$$

(ii) There is no ohmic resistance in the circuit.

$\therefore$ Average power consumed per cycle = **0**.

3. $X_L = 2\pi f L = 2 \times 3.14 \times 50 \times 6 = 1884\,\Omega$

$$Z = \sqrt{R^2 + X_L^2} = \sqrt{100^2 + 1884^2} = 1886.7\,\Omega$$

$$I_{rms} = \frac{\mathcal{E}_{rms}}{Z} = \frac{250}{1886.7} = \mathbf{0.1325\ A}$$

Power factor, $\cos\phi = \frac{R}{Z} = \frac{100}{1886.7} = \mathbf{0.053}$.

4. $X_L = 2\pi f L = 2 \times 3.14 \times 50 \times 10 = 3140\,\Omega$

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2 \times 3.14 \times 50 \times 2 \times 10^{-6}} = 1592\,\Omega$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{50^2 + (3140 - 1592)^2} = 1549\,\Omega$$

$$I_{rms} = \frac{\mathcal{E}_{rms}}{Z} = \frac{250}{1549} = \mathbf{0.16\ A}$$

Power factor, $\cos\phi = \frac{R}{Z} = \frac{50}{1549} = \mathbf{0.032.}$

5. $\mathcal{E} = \sqrt{V_R^2 + (V_L - V_C)^2} = \sqrt{80^2 + (100 - 40)^2} = \mathbf{100\ V}$

As $\tan\phi = \frac{V_L - V_C}{V_R}$

$$\therefore \quad \cos\phi = \frac{V_R}{\sqrt{V_R^2 + (V_L - V_C)^2}} = \frac{80}{100} = \mathbf{0.8.}$$

6. $V_{rms} = 0.707\,V_0 = \frac{1}{\sqrt{2}} \times 141.4 = \frac{141.4}{1.414} = 100\text{ V}$

$$I_{rms} = \frac{P_{av}}{V_{rms}} = \frac{300}{100} = \mathbf{3\ A.}$$

7. $\mathcal{E}_0 = 110\text{ V},\ I_0 = 5\text{ A}$

$$\phi = \omega t + \frac{\pi}{6} - \left(\omega t - \frac{\pi}{6}\right) = \frac{\pi}{3}$$

$$Z = \frac{\mathcal{E}_0}{I_0} = \frac{110}{5} = \mathbf{22\,\Omega}$$

$$P_{av} = \frac{\mathcal{E}_0 I_0}{2}\cos\phi = \frac{110 \times 5}{2} \times \cos\frac{\pi}{3} = \mathbf{137.5\ W.}$$

8. (i) Power factor will be unity at resonance, because then $Z = R$ and $\cos\phi = \frac{R}{Z} = 1$

$$\therefore\ f_r = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{200 \times 10^{-3} \times 500 \times 10^{-6}}}\text{ Hz}$$

$$= \frac{1}{2\pi \times 10^{-2}} = \frac{\mathbf{50}}{\pi}\ \mathbf{Hz}$$

(ii) $I_0 = \frac{\mathcal{E}_0}{R} = \frac{\sqrt{2}\,\mathcal{E}_{rms}}{R} = \frac{1.414 \times 100}{10} = \mathbf{14.14\ A.}$

(iii) Q-factor $= \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{10}\sqrt{\frac{200 \times 10^{-3}}{500 \times 10^{-6}}} = \frac{20}{10} = \mathbf{2.}$

9. Power factor, $\cos\phi = \frac{R}{Z} = 1$ or $Z = R$

$\therefore\ X_L = X_C$ or $2\pi f L = \frac{1}{2\pi f C}$

or $L = \frac{1}{4\pi^2 f^2 C} = \frac{1}{4 \times 9.87 \times (50)^2 \times 100 \times 10^{-6}}\text{ H}$

$= 0.1013\text{ H} = \mathbf{101.3\ mH}$

Current amplitude,

$$I_0 = \frac{\mathcal{E}_0}{R} = \frac{\sqrt{2}\,\mathcal{E}_{rms}}{R} = \frac{1.414 \times 200}{10} = \mathbf{28.28\ A.}$$

10. Clearly, $\omega = 1000\text{ rad s}^{-1}$

$\therefore\quad X_L = \omega L = 1000 \times 100 \times 10^{-3} = 100\,\Omega$

$$X_C = \frac{1}{\omega C} = \frac{1}{1000 \times 2 \times 10^{-6}} = 500\,\Omega$$

As $X_C > X_L$, the current leads the voltage by phase angle ϕ given by

$$\tan\phi = \frac{X_C - X_L}{R} = \frac{500 - 100}{400} = 1 \quad \therefore \quad \phi = \mathbf{45^\circ}$$

Thus the phase angle is 45° with the current leading the voltage. The power factor will be unity when $\phi = 0°$. Then new capacitance C' should be such that

$$X_{C'} = X_L = 100\,\Omega$$

or $\frac{1}{\omega C'} = 100$ or $C' = \frac{1}{\omega \times 100} = \frac{1}{1000 \times 100}\text{ F} = 10\mu\text{F}$

Clearly, we need to connect an additional capacitor of $(10 - 2) = 8\,\mu$F in parallel with the given capacitor for making power factor equal to unity.

11. $X_L = 2\pi f L = 2 \times 3.14 \times 50 \times 80 \times 10^{-3} = 25.13\,\Omega$

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2 \times 3.14 \times 50 \times 60 \times 10^{-6}} = 53.05\,\Omega$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{15^2 + (25.13 - 53.05)}$$

$= 31.7\,\Omega$

$$I_{rms} = \frac{\mathcal{E}_{rms}}{Z} = \frac{230}{31.7} = 7.26\,\Omega$$

$$P_{av}^R = I_{rms}^2 R = (7.26)^2 \times 15 = \mathbf{796.6\ W.}$$

$$P_{av}^L = V_{rms}^L\, I_{rms} \cos\frac{\pi}{2} = \mathbf{0.}$$

$$P_{av}^C = V_{rms}^C\, I_{rms} \cos\left(-\frac{\pi}{2}\right) = \mathbf{0.}$$

$$P_{av}^{total} = 790.6 + 0 + 0 = \mathbf{790.6\ W.}$$

12. (i) X is an inductor and Y is a capacitor.

(ii) In case of LR-circuit, $\cos\frac{\pi}{4} = \frac{R}{Z}$

$$\Rightarrow \quad \frac{1}{\sqrt{2}} = \frac{R}{Z} \quad \Rightarrow \quad Z = \sqrt{2}R$$

$\therefore \quad X_L = \sqrt{Z^2 - R^2} = \sqrt{2R^2 - R^2} = R$

In case of *CR*-circuit, $\cos\frac{\pi}{4} = \frac{R}{Z} = \frac{1}{\sqrt{2}}$

$\Rightarrow \quad Z = \sqrt{2}R$

$\therefore \quad X_C = \sqrt{Z^2 - R^2} = \sqrt{2R^2 - R^2} = R$

Clearly, $X_L = X_C = R$

The circuit is purely resistive. The impedance is minimum and current is maximum. Hence the power dissipated in the circuit is maximum.

7.21 *LC*-OSCILLATIONS

28. *What are LC-oscillations ? Explain qualitatively, how these oscillations are produced.*

LC-Oscillations. *When a charged capacitor is allowed to discharge through a non-resistive inductor, electrical oscillations of constant amplitude and frequency are produced. These oscillations are called LC-oscillations.*

Qualitative explanation for the production of LC-oscillations. Fig. 7.47(*a*) shows a capacitor with initial charge q_0, connected to an ideal inductor. The electrical energy stored in the charged capacitor is $U_E = \frac{1}{2}\frac{q_0^2}{C}$. As there is no current in the circuit, the energy stored in the magnetic field of the inductor is zero.

As the circuit is closed [Fig. 7.47(*b*)], the capacitor begins to discharge itself through the inductor, causing a current *I*. As the current *I* increases, it builds up magnetic field around the inductor. A part of electric energy of the capacitor gets stored in the inductor in the form of magnetic energy, $U_B = \frac{1}{2}LI^2$

At a later instant [Fig. 7.47(*c*)], the capacitor gets fully discharged and p.d. across its plates becomes zero. The current reaches its maximum value I_0, the energy stored in the magnetic field is $\frac{1}{2}LI_0^2$. Thus the entire electrostatic energy of the capacitor has been converted into the magnetic field energy of the inductor.

After the discharge of the capacitor is complete, the magnetic flux linked with the inductor decreases, inducing a current in the same direction (Lenz's law) as the earlier current, as shown in Fig. 7.47(*d*). The current thus persists, though with decreasing magnitude, and charges the capacitor in the opposite direction. The magnetic energy of the inductor begins to change into the electrostatic energy of the capacitor. This process continues till the capacitor is fully charged [Fig. 7.47(*e*)]. But it is charged with a polarity opposite to that in its initial state [Fig. 7.47(*a*)]. Thus the entire energy is again stored as $\frac{1}{2}\frac{q_0^2}{C}$ in the electric field of the capacitor.

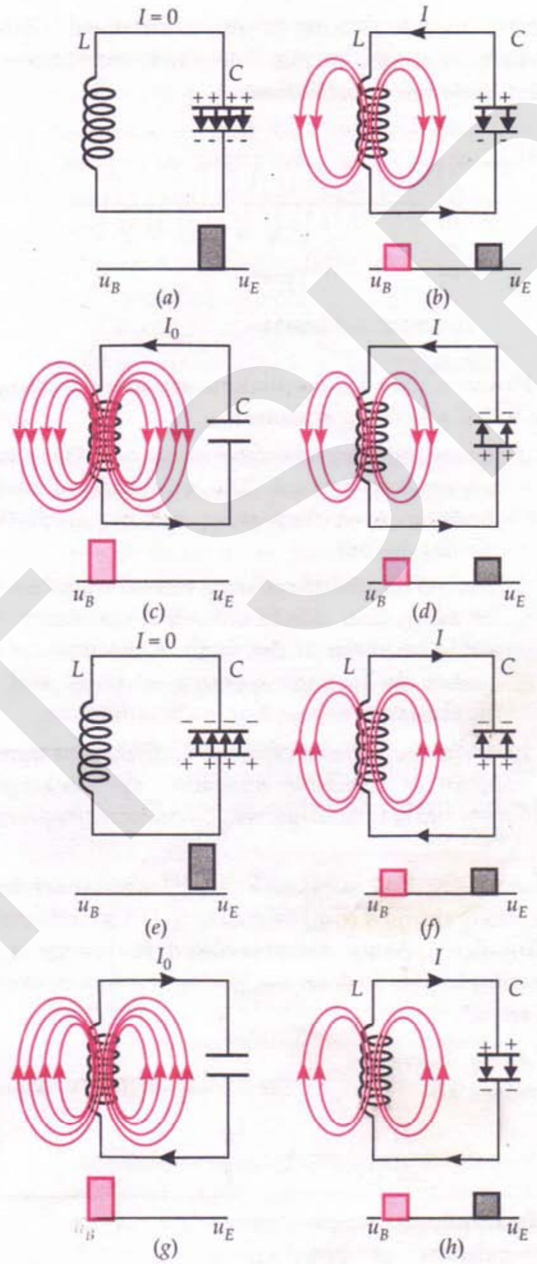


Fig. 7.47 The oscillations in an *LC*-circuit.

The capacitor begins to discharge again, sending current in opposite direction [Fig. 7.47(*f*)]. The energy is once again transferred to the magnetic field of the inductor. Thus the process repeats in the opposite direction [Fig. 7.47(*g*) and (*h*)]. The circuit eventually returns to the initial state [Fig. 7.47(*a*)].

Thus the energy of system continuously surges back and forth between the electric field of the capacitor and the magnetic field of the inductor. This produces electrical oscillations of a definite frequency f_0. These are called *LC-oscillations*. If there is no loss of energy, the amplitude of the oscillations remains constant as shown in Fig. 7.48. Such oscillations are called ***undamped oscillations***.

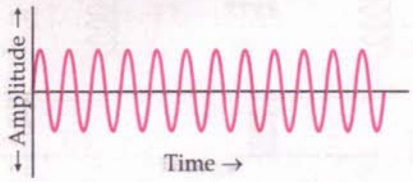


Fig. 7.48 Undamped oscillations.

However, the LC-oscillations are usually damped due to the following reasons :

1. Every inductor has some resistance. This causes energy loss as heat. The amplitude of oscillations goes on decreasing and the oscillations finally die out.
2. Even if the resistance were zero, the total energy of the system would not remain constant. It is radiated away in the form of electromagnetic waves. In fact the working of radio and TV transmitters is based on such radiations.

29. *Show that when a capacitor is discharged through an inductor of negligible resistance, the discharge is oscillatory and simple harmonic. Calculate its frequency.*

Mathematical treatment of LC-oscillations. As shown in Fig. 7.49, suppose a capacitor of capaci- tance C, initially charged to q_0, be connected to an inductor of inductance L. As the circuit is closed, the charge on the capacitor begins to decrease, giving rise to a current in the circuit.

As q decreases, I increases, so

$$I = -\frac{dq}{dt}$$

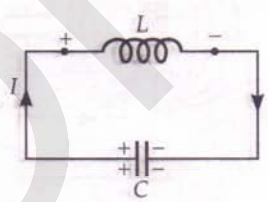


Fig. 7.49 Polarity of induced emf across L as I increases in the circuit.

Induced emf across the inductor at any instant

$$= -L\frac{dI}{dt}$$

P.D. across the capacitor at that instant $= \frac{q}{C}$

According to Kirchhoff's loop rule,

$$-L\frac{dI}{dt} + \frac{q}{C} = 0$$

or $$+L\frac{d^2q}{dt^2} + \frac{q}{C} = 0 \qquad \left[\because I = -\frac{dq}{dt}, \frac{dI}{dt} = -\frac{d^2q}{dt^2}\right]$$

or $$\frac{d^2q}{dt^2} + \frac{1}{LC}q = 0$$

or $$\frac{d^2q}{dt^2} + \omega_0^2 q = 0$$

where $\omega_0 = \frac{1}{\sqrt{LC}}$. This is a linear differential equation of second order. It has a general solution of the form :

$$q = A\cos\omega_0 t + B\sin\omega_0 t$$

At $t = 0$, $q = q_0$, so

$$q_0 = A\cos 0 + B\sin 0$$

or $A = q_0$

Also , $$\frac{dq}{dt} = -\omega_0 A\sin\omega_0 t + \omega_0 B\cos\omega_0 t$$

At $t = 0$, $q = q_0$ (maximum) and so $\frac{dq}{dt} = 0$

$\therefore$ $0 = -\omega_0 A\sin 0 + \omega_0 B\cos 0$

or $B = 0$

Hence $q = q_0\cos\omega_0 t$

and $$I = -\frac{dq}{dt} = \omega_0 q_0 \sin\omega_0 t$$

Thus the charge on the capacitor plates oscillates simple harmonically with time with angular frequency,

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

Frequency of oscillation, $f_0 = \frac{\omega_0}{2\pi} = \frac{1}{2\pi\sqrt{LC}}$.

30. *Show that in the free oscillations of an LC circuit, the sum of energies stored in capacitor and the inductor is constant in time.* [NCERT]

Conservation of Energy In Lc-Oscillations. Just as the sum of potential and kinetic energies remains constant in SHM, the sum of energies stored in the capacitor and inductor remains constant during LC-oscillations. At any instant, the electrostatic energy stored in the capacitor is

$$U_E = \frac{1}{2}\cdot\frac{q^2}{C}$$

The magnetic energy stored in the inductor at any instant is

$$U_B = \frac{1}{2}LI^2$$

If there is no (resistive) loss of energy, then the total energy of the LC-circuit at any instant will be

$$U = U_E + U_B = \frac{1}{2}\cdot\frac{q^2}{C} + \frac{1}{2}LI^2$$

But $q = q_0\cos\omega_0 t$ and $I = -\frac{dq}{dt} = \omega q_0 \sin\omega_0 t$

$$\therefore\ U = \frac{1}{2C}\cdot q_0^2 \cos^2 \omega_0 t + \frac{1}{2} L\omega_0^2 q_0^2 \sin^2 \omega_0 t$$

$$= \frac{1}{2C}\cdot q_0^2 \cos^2 \omega_0 t + \frac{1}{2} L \frac{1}{LC} q_0^2 \sin^2 \omega_0 t$$

$$= \frac{1}{2}\cdot\frac{q_0^2}{C}[\cos^2 \omega_0 t + \sin^2 \omega_0 t] = \frac{1}{2}\cdot\frac{q_0^2}{C}$$

$$= \frac{1}{2} CV_0^2 = \text{Initial energy, as expected}$$

31. *Give and explain the mechanical analogy of LC-oscillations.*

Mechanical analogy for *LC*-oscillations. The *LC*-oscillations are similar to the oscillations of a mass-spring system. In the *LC*-system, the energy alternates between electrostatic and magnetic forms while in the mass-spring system, it alternates between potential and kinetic forms. The capacitor acts like a spring while the inductor acts like an inertial mass. The charge corresponds to displacement and the current corresponds to velocity.

The displacement x of oscillating mass satisfies the differential equation :

$$\frac{d^2x}{dt^2} + \omega_0^2 x = 0, \quad \text{where} \quad \omega_0 = \sqrt{\frac{k}{m}}$$

The charge q of the capacitor satisfies a similar equation :

$$\frac{d^2q}{dt^2} + \omega_0^2 q = 0, \quad \text{where} \quad \omega_0 = \frac{1}{\sqrt{LC}}$$

Clearly, x corresponds to q.

For a mechanical system,

$$F = m\frac{dv}{dt} = m\frac{d^2x}{dt^2}$$

For an electrical system,

$$\mathcal{E} = -L\frac{dI}{dt} = -L\frac{d^2q}{dt^2}$$

On comparing the above two equations, we see that L is analogous to mass m *i.e.*, L is a measure of resistance to change in current.

Again, in case of LC system, $\omega_0 = \frac{1}{\sqrt{LC}}$

In case of mass-spring system, $\omega_0 = \sqrt{\frac{k}{m}}$

Obviously, $\frac{1}{C}$ is analogous to k. The force constant k is the force required to produce unit displacement while $\frac{1}{C}\left(=\frac{V}{q}\right)$ is the potential difference required to store a unit charge.

Table 7.1 Analogies between Mechanical and Electrical Quantities

Mechanical System	Electrical System
Mass m	Inductance L
Displacement x	Charge q
Velocity $v = \frac{dx}{dt}$	Current $I = \frac{dq}{dt}$
Force constant k	Reciprocal capacitance $\frac{1}{C}$
Mechanical energy $E = \frac{1}{2}kx^2 + \frac{1}{2}mv^2$	Electromagnetic energy $U = \frac{1}{2}\frac{q^2}{C} + \frac{1}{2}LI^2$

Examples based on LC-Oscillations

Formulae Used

1. Angular frequency of free oscillations of an *LC*-circuit,
$$\omega = \frac{1}{\sqrt{LC}}$$
2. Frequency of free oscillations of an *LC*-circuit,
$$f = \frac{1}{2\pi\sqrt{LC}}$$
3. Instantaneous charge on the capacitor,
$$q = q_0 \cos \omega t$$
4. Instantaneous current in the *LC*-circuit,
$$I = -\frac{dq}{dt} = I_0 \sin \omega t, \quad \text{where } I_0 = \omega q_0$$
5. Electrical energy stored in the capacitor at any instant,
$$U_E = \frac{1}{2}\cdot\frac{q^2}{C}$$
$$U_E^{\max} = \frac{1}{2}\cdot\frac{q_0^2}{C}$$
6. Magnetic energy stored in the inductor at any instant,
$$U_B = \frac{1}{2}LI^2$$
$$U_B^{\max} = \frac{1}{2}LI_0^2$$
7. Total energy stored in the *LC*-circuit,
$$U = U_E + U_B = \frac{1}{2}\cdot\frac{q_0^2}{C} = \frac{1}{2}LI_0^2.$$

Units Used

Charges q and q_0 are in coulomb, currents I and I_0 in ampere, inductance L in henry, capacitance C in farad, angular frequency ω in rad s^{-1}, and energies U, U_E and U_B are in joule.

Example 65. *Calculate the wavelength of radiowaves radiated out by a circuit consisting of 0.02 μF capacitor and 8 μH inductor in series.*

Solution. Here $C = 0.02\ \mu F = 0.02 \times 10^{-6}$ F,
$L = 8\ \mu H = 8 \times 10^{-6}$ H

$$f = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{0.02\times10^{-6}\times 8\times10^{-6}}}$$

$$= 3.98 \times 10^5 \text{ Hz}$$

The wavelength of the radiowaves produced is

$$\lambda = \frac{c}{f} = \frac{3\times10^8}{3.98\times10^5} = \mathbf{7.54\times10^2\ m}.$$

Example 66. *An inductor of inductance 2.0 mH is connected across a charged capacitor of capacitance 5.0 μF and the resulting LC-circuit is set oscillating at its natural frequency. Let q denote the instantaneous charge on the capacitor and I the current in the circuit. It is found that maximum value of charge q is 200 μC.*

(*a*) *When $q = 100\ \mu C$, what is the value of $\frac{dI}{dt}$?*

(*b*) *When $q = 200\ \mu C$, what is the value of I ?*

(*c*) *Find the maximum value of I.*

(*d*) *When I is equal to one-half its maximum value, what is the value of q ?* [IIT 1998]

Solution. Here $L = 2.0$ mH $= 2.0 \times 10^{-3}$ H
$C = 5.0\ \mu F = 5.0 \times 10^{-6}$ F

The natural frequency of LC-oscillations is given by

$$\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{2.0\times10^{-3}\times5.0\times10^{-6}}}$$

$$= 10^4 \text{ rad s}^{-1}$$

The charge on the capacitor at any instant t during LC-oscillations,

$$q = q_0 \cos \omega t$$

$$\therefore \quad I = -\frac{dq}{dt} = q_0\,\omega \sin \omega t = I_0 \sin \omega t$$

and $$\frac{dI}{dt} = q_0\,\omega^2 \cos \omega t = \omega^2 q$$

(*a*) When $q = 100\ \mu C = 10^{-4}$ C,

$$\frac{dI}{dt} = \omega^2 q = 10^8 \times 10^{-4} = \mathbf{10^4\ As^{-1}}.$$

(*b*) Here $q = 200\ \mu C$. Also $q_0 = 200\ \mu C$

As $q = q_0 \cos \omega t \quad \therefore \quad 200 = 200 \cos \omega t$

or $\cos \omega t = 1$ or $\omega t = 0°$

$$\therefore \quad I = q_0\,\omega \sin 0° = \mathbf{0}.$$

This can also be followed from the fact that when $q = 200\ \mu C = q_0$, the capacitor is fully charged. At this instant, the current in the LC-circuit is zero.

(*c*) $I_0 = \omega q_0 = 10^4 \times 200 \times 10^{-6} = \mathbf{2.0\ A}$.

(*d*) $I = I_0 \sin \omega t$

When $I = I_0/2$, we have

$$\frac{I_0}{2} = I_0 \sin \omega t \quad \text{or} \quad \sin \omega t = 0.5$$

$$\therefore \quad \omega t = 30°$$

Here $q = q_0 \cos \omega t = 200 \times 10^{-6} \times \cos 30°$
$= 200 \times 10^{-6} \times 0.866$ C
$= 173.2 \times 10^{-6}$ C $= \mathbf{173.2\ \mu C}$

Problems For Practice

1. A coil of inductance 150 mH is connected in series with a variable capacitor of capacitance 20 pF to 500 pF. Calculate the frequency range over which the circuit can be tuned. (Ans. 1.84×10^4 Hz to 9.2×10^4 Hz)
2. A 10 μF capacitor is charged to a potential of 25 V. The battery is then disconnected and pure 100 mH coil is connected across the capacitor so that LC-oscillations are set up. Calculate the maximum current in the coil. (Ans. 0.25 A)
3. A 1.5 mH inductor in an LC-circuit stores a maximum energy of 30 μJ. What is the maximum current in the circuit ? (Ans. 0.2 A)
4. In an oscillatory circuit, the self-inductance of the coil used is 10 mH. If the oscillatory frequency of the circuit is 1.0 MHz, find the capacitance of the capacitor connected in the circuit. (Ans. 2.53 pF)
5. A wave of wavelength 300 m can be radiated through a transmitter. A capacitor of capacitance 2.4 μF is available. What is the inductance of the coil required for the oscillatory circuit ? (Ans. 1.056×10^{-8} H)

HINTS

1. $$f_1 = \frac{1}{2\pi\sqrt{LC_1}} = \frac{1}{2\pi\sqrt{150\times10^{-3}\times20\times10^{-12}}}$$
$$= \mathbf{9.2\times10^4\ Hz}$$
$$f_2 = \frac{1}{2\pi\sqrt{LC_2}} = \frac{1}{2\pi\sqrt{150\times10^{-3}\times500\times10^{-12}}}$$
$$= \mathbf{1.84\times10^4\ Hz}$$

2. Maximum magnetic energy in the inductor
= Maximum electric energy in the capacitor
$$\frac{1}{2}LI_0^2 = \frac{1}{2}CV^2$$
$$\therefore \quad I_0 = \sqrt{\frac{C}{L}}\cdot V = \sqrt{\frac{10\times10^{-6}}{100\times10^{-3}}}\times 25$$
$$= 10^{-2}\times 25 = \mathbf{0.25\ A}.$$

3. $\frac{1}{2} LI_0^2$ = Maximum energy in the inductor

$$\frac{1}{2} \times 1.5 \times 10^{-3} \times I_0^2 = 30 \times 10^{-6} \text{ J}$$

$$I_0^2 = \frac{60 \times 10^{-6}}{1.5 \times 10^{-3}} = 4 \times 10^{-2}$$

$$I_0 = 2 \times 10^{-1} = \mathbf{0.2\ A}$$

4. $$C = \frac{1}{4\pi^2 f^2 L} = \frac{1}{4 \times 9.87 \times (10^6)^2 \times 10 \times 10^{-3}}$$

$$= 2.53 \times 10^{-12} \text{ F} = \mathbf{2.53\ pF}.$$

5. $$f = \frac{c}{\lambda} = \frac{1}{2\pi\sqrt{LC}} \quad \text{or} \quad \frac{c^2}{\lambda^2} = \frac{1}{4\pi^2 LC}$$

or $$L = \frac{\lambda^2}{4\pi^2 c^2 C} = \frac{(300)^2}{4 \times 9.87 \times (3 \times 10^8)^2 \times 2.4 \times 10^{-6}}$$

$$= \mathbf{1.056 \times 10^{-8}\ H}.$$

7.22 TRANSFORMER

32. *What is a transformer ? Explain the principle, construction, working and theory of a transformer. How is current affected in a transformer ? What are the various energy losses in a transformer ? How can they be reduced ?*

Transformer. *A transformer is an electrical device for converting an alternating current at low voltage into that at high voltage or vice versa.* If it increases the input voltage, it is called ***step up transformer*** and if it decreases the input voltage, it is called ***step down transformer***.

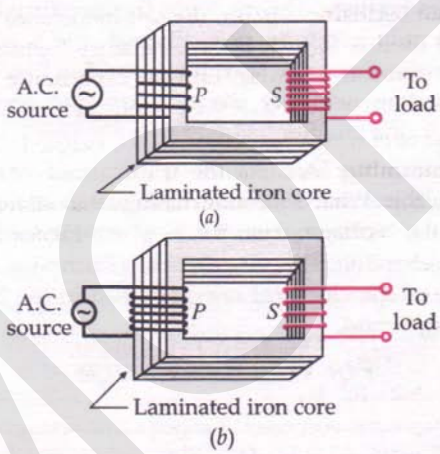


Fig. 7.50 (*a*) Step-up and (*b*) step-down, transformer.

Principle. *It works on the principle of mutual induction, i.e., when a changing current is passed through one of the two inductively coupled coils, an induced emf is set up in the other coil.*

Construction. A transformer essentially consists of two coils of insulated copper wire having different number of turns and wound on the same soft iron core. The coil *P* to which electric energy is supplied is called the ***primary*** and the coil from which energy is drawn or output is obtained is called the ***secondary.*** To prevent energy losses due to eddy currents, a laminated core is used. Because of high permeability of soft iron, the entire magnetic flux due to the current in the primary coil practically remains in the iron core and hence passes fully through the secondary. This also prevents the stray currents being generated in the conductors lying around and the consequent power loss.

Two types of arrangements are generally used for winding of primary and secondary coils in a transformer :

1. Core type. In the core type transformers, the primary and secondary coils are wound on separate limbs of the core so that the core is largely surrounded by the coils. Many of the modern transformers are of closed core type as shown in Fig. 7.51(*a*)

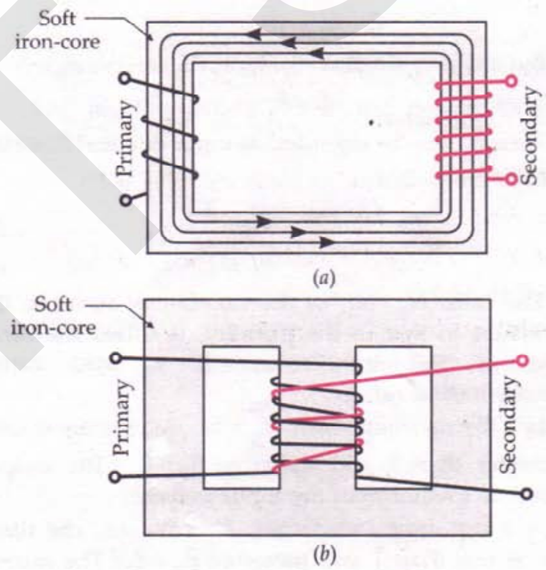


Fig. 7.51 (*a*) Core type and (*b*) Shell type transformer.

2. Shell type. In the shell type transformers, the primary and secondary coils are wound one over another on the same limb of the iron core. The coils are very largely surrounded by the iron core. Transformers used in radio and TV transmitters and receivers are of shell type, as shown in Fig. 7.48(*b*).

Working. As the alternating current flows through the primary, it generates an alternating magnetic flux in the core which also passes through the secondary. This changing flux sets up an induced emf in the secondary, also a self-induced emf in the primary. If there is no leakage of magnetic flux, then flux linked with each turn of the primary will be equal to that linked with each turn of the secondary.

Theory. Consider the situation when no load is connected to the secondary, *i.e.*, its terminals are open. Let N_1 and N_2 be the number of turns in the primary and secondary respectively. Then

Induced emf in the primary coil, $\varepsilon_1 = -N_1 \frac{d\phi}{dt}$

Induced emf in the secondary coil, $\varepsilon_2 = -N_2 \frac{d\phi}{dt}$

where ϕ is the magnetic flux linked with each turn of the primary or secondary at any instant. Thus

$$\frac{\varepsilon_2}{\varepsilon_1} = \frac{N_2}{N_1}$$

Let ε be the emf applied to the primary. By Lenz's law, self-induced emf ε_1 opposes ε in the primary coil.

$\therefore$ Resultant emf in the primary $= \varepsilon - \varepsilon_1$

This emf sends current I_1, through the primary coil of resistance R.

$$\therefore \qquad \varepsilon - \varepsilon_1 = RI_1$$

But R is very small, so the term RI_1 can be neglected.

Then $\qquad \varepsilon = \varepsilon_1$

Thus ε_1 may be regarded as input emf and ε_2 as the output emf.

$$\frac{\varepsilon_2}{\varepsilon_1} = \frac{\text{Output emf}}{\text{Input emf}} = \frac{N_2}{N_1} \qquad ...(1)$$

The ratio N_2 / N_1, of the number of turns in the secondary to that in the primary, is called the ***turns ratio*** of the transformer. It is also called ***transformation ratio***.

In a step up transformer, $N_2 > N_1$, *i.e.*, the turns ratio is greater than 1 and therefore $\varepsilon_2 > \varepsilon_1$. The output voltage is greater than the input voltage.

In a step down transformer, $N_2 < N_1$, *i.e.*, the turns ratio is less than 1 and therefore $\varepsilon_2 < \varepsilon_1$. The output voltage is less than the input voltage.

It may be noted that equation (1) has been derived by using the following three assumptions :

1. The primary resistance and current are small.
2. The same flux links both with the primary and secondary windings as the flux leakage from the core is negligibly small.
3. The terminals of the secondary are open or the current taken from it is small.

Currents in primary and secondary. Assuming the transformer to be ideal one so that there are no energy losses, then

Input power = Output power or $\varepsilon_1 I_1 = \varepsilon_2 I_2$

where I_1 and I_2 are the currents in the primary and secondary, respectively.

Hence
$$\frac{I_1}{I_2} = \frac{\varepsilon_2}{\varepsilon_1} = \frac{N_2}{N_1} \qquad ...(2)$$

Thus a step up transformer steps up the voltage, but steps down the current exactly in the same ratio. Similarly, a step down transformer steps down the voltage but steps up the current exactly in the same ratio.

The efficiency of a transformer is defined as

$$\eta = \frac{\text{Power output}}{\text{Power input}} \times 100\% \qquad ...(3)$$

The efficiency of real transformers is fairly high (90 – 99%) though not 100%.

Energy losses in transformers. The main causes for energy loss in transformers are as follows :

1. Copper loss. Some energy is lost due to heating of copper wires used in the primary and secondary windings. This power loss ($= I^2R$) can be minimised by using thick copper wires of low resistance.

2. Eddy current loss. The alternating magnetic flux induces eddy currents in the iron core which leads to some energy loss in the form of heat. This loss can be reduced by using laminated iron core.

3. Hysteresis loss. The alternating current carries the iron core through cycles of magnetisation and demagnetisation. Work is done in each of these cycles and is lost as heat. This is called hysteresis loss and can be minimised by using core material having narrow hysteresis loop.

4. Flux leakage. The magnetic flux produced by the primary may not fully pass through the secondary. Some of the flux may leak into air. This loss can be minimised by winding the primary and secondary coils over one another.

5. Humming loss. As the transformer works, its core lengthens and shortens during each cycle of the alternating voltage due to a phenomenon called ***magnetostriction***. This gives rise to a humming sound. So some of the electrical energy is lost in the form of humming sound.

For Your Knowledge

- A step-up transformer changes a low-voltage into a high-voltage. This does not violate the law of conservation of energy. The current decreases by the same proportion. When voltage increases n times, the current reduces to $1/n$ times.
- A transformer is essentially an a.c. device. It cannot work on d.c. It changes alternating voltages/currents. It does not affect the frequency of a.c.
- The small transformers are self-cooled, which transfer heat directly to the surroundings. Large transformers are cooled by placing them in oil tanks to prevent overheating.

7.23 USES OF TRANSFORMERS

33. *State some of the important uses of transformers.*

Some uses of transformers :

1. Small transformers are used in radio receivers, telephones, loud speakers, etc.
2. In voltage regulators for TV, refrigerators, air-conditioners, computers, etc.
3. In stabilised power supplies.
4. A step-down transformer is used for obtaining large current for electric welding.
5. A step-down transformer is used in induction furnace for melting metals.
6. A step-up transformer is used for the production of *X*-rays.
7. In the transmission of electric energy from the generating stations to the consumers.

7.24 LONG DISTANCE TRANSMISSION OF ELECTRICAL POWER

34. *Explain the use of transformers in long distance transmission of electric power.*

Use of transformers in long distance transmission of electric power. The most important application of transformers is in the transmission of electrical power from a power station to far away areas where it is actually used. Following are the disadvantages of transmitting the electrical power at low voltage :

1. Large length of transmission cables have appreciable resistance. Hence a large amount of energy (I^2Rt) will be lost as heat during transmission.

2. Large voltage drop (IR) occurs along the line wire. Hence the voltage at the receiving station will be much smaller than that at the generating station.

3. To carry large currents and to keep the resistance of transmission wires low, thick wires have to be used. The cost of installing thick wires will be extremely high.

Thus the long distance power transmission at low voltage and high current is neither efficient nor economical. If I is the current in the cable, and R its resistance, the power wasted in the cable is I^2R. The power loss can be reduced by reducing I or R. The power supplied by the generator is given by $P = VI$, where V is the voltage across its terminals. Since $I = P/V$, for a given amount of power P, the power loss is less if I is less or V is high.

In actual practice, as shown in Fig. 7.52, a typical power station generates 1000 kW at 6600 volts. This voltage is first stepped up to 132000 volts before transmission. Transmission lines from different power stations in a region deliver power to a common regional pool, called the *grid*. From the grid, the power is fed to the cities at 33000 V, the stepping down is done outside the city. Then again at a sub-station, the supply is stepped down to 6600 V. For domestic purposes, the voltage is again stepped down to 220 V.

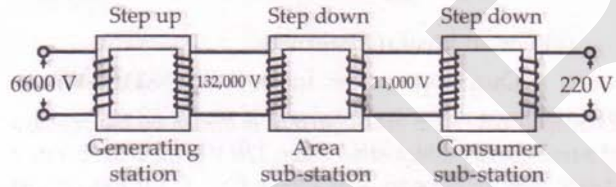


Fig. 7.52 Schematic diagram of a distribution system.

Examples Based on Transformers and Long Distance Power Transmission

Formulae Used

1. The voltages and currents in a transformer are related as

$$\frac{\mathcal{E}_2}{\mathcal{E}_1} = \frac{I_1}{I_2} = \frac{N_2}{N_1} = k$$

where suffix 1 refers to primary coil, 2 to secondary coil and k is the transformation or turns ratio.

2. $\mathcal{E}_1 I_1$ (Power in primary coil) $= \mathcal{E}_2 I_2$ (Power in secondary coil)

3. Efficiency of a transformer,

$$\eta = \frac{\text{Output power}}{\text{Input power}} \times 100\%$$

4. Power is transmitted from power stations to sub-stations at very high voltages to reduce cost and reduce losses.

Units Used

Voltages $\mathcal{E}_1, \mathcal{E}_2$ are in volts ; currents I_1, I_2 in ampere ; and k and η have no units.

Example 67. *The primary coil of an ideal step-up transformer has 100 turns and the transformation ratio is also 100. The input voltage and the power are 220 V and 1100 W respectively. Calculate :*

(*i*) *number of turns in the secondary*
(*ii*) *the current in the primary*
(*iii*) *voltage across the secondary*
(*iv*) *the current in the secondary*
(*v*) *power in the secondary.* [CBSE D 06]

Solution. Here $N_1 = 100$, $\mathcal{E}_1 = 220$ V, $P_1 = 1100$ W

(*i*) Transformation ratio, $k = \frac{N_2}{N_1} = 100$

$\therefore\ N_2 = 100\ N_1 = 100 \times 100 =$ **10,000.**

(*ii*) $I_1 = \frac{P_1}{\mathcal{E}_1} = \frac{1100}{220} = \mathbf{5\ A.}$

(*iii*) $\mathcal{E}_2 = k\mathcal{E}_1 = 100 \times 220 = \mathbf{22{,}000\ V.}$

(*iv*) $I_2 = \frac{I_1}{k} = \frac{5}{100} = \mathbf{0.05\ A.}$

(*v*) For an ideal transformer

Output power = Input power = **1100 W.**

Example 68. *How much current is drawn by the primary of a transformer which steps down 220 V to 22 V to operate a device with an impedance of 220 Ω ?* **[CBSE OD 08]**

Solution. Here $\mathcal{E}_1 = 220\ \text{V}, \mathcal{E}_2 = 22\ \text{V}, Z_2 = 220\ \Omega$

Current drawn by the secondary or by the device of impedance 220 Ω is

$$I_2 = \frac{\mathcal{E}_2}{Z_2} = \frac{22}{220} = 0.1\ \text{A}$$

If there are no energy losses, then

Input power = Output power *i.e.*, $\mathcal{E}_1 I_1 = \mathcal{E}_2 I_2$

or $$I_1 = \frac{\mathcal{E}_2 I_2}{\mathcal{E}_1} = \frac{22 \times 0.1}{220} = \mathbf{0.01\ A.}$$

Example 69. *A transformer has 500 turns in the primary and 1000 turns in its secondary winding. The primary voltage is 200 V and the load in the secondary is 100 Ω. Calculate the current in the primary, assuming it to be an ideal transformer.* **[ISCE 02]**

Solution. Here $N_1 = 500,\ N_2 = 1000,\ \mathcal{E}_1 = 200\ \text{V}, R_2 = 100\ \Omega$

$$\mathcal{E}_2 = \frac{N_2}{N_1} \cdot \mathcal{E}_1 = \frac{1000}{500} \times 200 = 400\ \text{V}$$

Current in the secondary,

$$I_2 = \frac{\mathcal{E}_2}{R_2} = \frac{400}{100} = 4\ \text{A}$$

For an ideal transformer,

$$\mathcal{E}_1 I_1 = \mathcal{E}_2 I_2$$

∴ Current in the primary,

$$I_1 = \frac{\mathcal{E}_2 I_2}{\mathcal{E}_1} = \frac{400 \times 4}{200} = \mathbf{8\ A.}$$

Example 70. *In an ideal transformer, number of turns in the primary and secondary are 200 and 1000 respectively. If the power input to the primary is 10 kW at 200 V, calculate (i) output voltage and (ii) current in primary.* **[CBSE D 02]**

Solution. Here $N_1 = 200,\ N_2 = 1000,\ \mathcal{E}_1 = 200\ \text{V}, P_1 = 10\ \text{kW} = 10{,}000\ \text{W}$

(*i*) As $\frac{N_2}{N_1} = \frac{\mathcal{E}_2}{\mathcal{E}_1}$

∴ Output voltage,

$$\mathcal{E}_2 = \frac{N_2}{N_1} \times \mathcal{E}_1 = \frac{1000}{200} \times 200 = \mathbf{1000\ V.}$$

(*ii*) Input power, $P_1 = I_1 \mathcal{E}_1$

∴ Current in primary,

$$I_1 = \frac{P_1}{\mathcal{E}_1} = \frac{10{,}000}{200} = \mathbf{50\ A.}$$

Example 71. *The output voltage of an ideal transformer, connected to a 240 V a.c. mains is 24 V. When this transformer is used to light a bulb with rating 24 V, 24 W, calculate the current in the primary coil of the circuit.* **[CBSE OD 2000]**

Solution. Here $\mathcal{E}_1 = 240\ \text{V},\ \mathcal{E}_2 = 24\ \text{V},\ P_2 = 24\ \text{W}$

Current in the secondary, $I_2 = \frac{P_2}{\mathcal{E}_2} = \frac{24}{24} = 1\ \text{A}$

For ideal transformer, $\mathcal{E}_1 I_1 = \mathcal{E}_2 I_2$

∴ Current in the primary,

$$I_1 = \frac{\mathcal{E}_2 I_2}{\mathcal{E}_1} = \frac{24 \times 1}{240} = \mathbf{0.1\ A.}$$

Example 72. *A transformer of 100% efficiency has 200 turns in the primary and 40,000 turns in the secondary. It is connected to a 220 V a.c. mains and the secondary feeds to a 100 resistance. Calculate the output potential difference per turn and the power delivered to the load.*

Solution. Here $N_1 = 200,\ N_2 = 40{,}000,\ \mathcal{E}_1 = 220\ \text{V}, R_2 = 100\ \text{k}\Omega = 10^5\ \Omega$

As $\frac{\mathcal{E}_2}{\mathcal{E}_1} = \frac{N_2}{N_1}$

$$\therefore \quad \mathcal{E}_2 = \frac{N_2}{N_1} \cdot \mathcal{E}_1 = \frac{40000}{200} \times 220 = \mathbf{44{,}000\ V}$$

Output p.d. per turn $= \frac{\mathcal{E}_2}{N_2} = \frac{44000}{40000} = \mathbf{1.1\ V}$

Power delivered to the load

$$= \mathcal{E}_2 I_2 = \frac{\mathcal{E}_2^2}{R_2} = \frac{(44000)^2}{10^5}$$

$$= 19360\ \text{W} = \mathbf{19.36\ kW.}$$

Example 73. *A step down transformer is used to reduce the main supply of 220 V to 11 V. If the primary draws a current of 5 A and the secondary 90 A, what is the efficiency of the transformer ?*

Solution. Here $\mathcal{E}_1 = 220$ V, $\mathcal{E}_2 = 11$ V, $I_1 = 5$ A, $I_2 = 90$ A

Power input $= \mathcal{E}_1 I_1 = 220 \times 5 = 1100$ W

Power output $= \mathcal{E}_2 I_2 = 11 \times 90 = 990$ W

Efficiency,

$$\eta = \frac{\text{Power output}}{\text{Power input}} = \frac{990}{1100} = 0.90 = \mathbf{90\%}.$$

Example 74. *Calculate the current drawn by the primary of a transformer, which steps down 200 V to 20 V to operate a device of resistance 20 Ω. Assume the efficiency of the transformer to be 80%.* [CBSE D 01]

Solution. Here $\mathcal{E}_1 = 20$ V, $\mathcal{E}_2 = 200$ V, $R_2 = 20\,\Omega$, $\eta = 80\%$

Current flowing through secondary,

$$I_2 = \frac{\mathcal{E}_2}{R_2} = \frac{20}{20} = 1 \text{ A}$$

$$\text{Efficiency, } \eta = \frac{\text{Power output}}{\text{Power input}} = \frac{\mathcal{E}_2 I_2}{\mathcal{E}_1 I_1}$$

$$\therefore \quad \frac{80}{100} = \frac{20 \times 1}{200 \times I_1}$$

or $\quad I_1 = \mathbf{0.125\ A}$.

Example 75. *A 10 kW transformer has 20 turns in the primary and 100 turns in the secondary circuit. An a.c. voltage $\mathcal{E}_1 = 600 \sin 314\, t$ is applied to the primary. Find (i) the maximum value of flux and (ii) the maximum value of the secondary voltage.*

Solution. (*i*) Flux linked with each turn of primary,

$$\phi = BA \cos \omega t = \phi_0 \cos \omega t$$

Here $\phi_0 = BA$ = maximum value of flux linked with each turn

$$\therefore \quad \mathcal{E}_1 = -N_1 \frac{d\phi}{dt} = -N_1 \frac{d}{dt}(\phi_0 \cos \omega t)$$

$$= \omega N_1 \phi_0 \sin \omega t$$

Peak value of $\mathcal{E}_1$

$$= \mathcal{E}_0 = \omega N_1 \phi_0 \quad \text{or} \quad \phi_0 = \frac{\mathcal{E}_0}{\omega N_1}$$

Given $\quad \mathcal{E}_1 = 600 \sin 314\, t = \mathcal{E}_0 \sin \omega t$

$\therefore \quad \mathcal{E}_0 = 600$ V, $\omega = 314$ rad s^{-1}

$$\text{Hence} \quad \phi_0 = \frac{600}{314 \times 20} = \mathbf{0.0955\ Wb.}$$

$$(ii) \quad \frac{\mathcal{E}_2^0}{\mathcal{E}_1^0} = \frac{N_2}{N_1}$$

$\therefore$ Maximum value of secondary voltage is

$$\mathcal{E}_2^0 = \frac{N_2}{N_1} \cdot \mathcal{E}_1^0 = \frac{100}{20} \times 600 = \mathbf{3000\ V.}$$

Example 76. (*i*) *The primary of a transformer has 400 turns while the secondary has 2000 turns. If the power output from the secondary at 1100 V is 12.1 kW, calculate the primary voltage. (ii) If the resistance of the primary is 0.2 Ω and that of the secondary is 2.0 Ω and the efficiency of the transformer is 90 %, calculate the heat losses in the primary and the secondary coils.*

Solution. (*i*) $N_1 = 400$, $N_2 = 2000$, $\mathcal{E}_2 = 1100$ V

$$\mathcal{E}_1 = \mathcal{E}_2 \cdot \frac{N_1}{N_2} = 1100 \times \frac{400}{2000} = \mathbf{220\ V}.$$

(*ii*) Resistance of primary, $R_1 = 0.2\,\Omega$

Resistance of secondary, $R_2 = 2.0\,\Omega$

Output power $= \mathcal{E}_2 I_2 = 12.1$ kW $= 12100$ W

$\therefore$ Current in the secondary,

$$I_2 = \frac{\mathcal{E}_2 I_2}{\mathcal{E}_2} = \frac{12100}{1100} = 11 \text{ A}$$

$$\text{As Efficiency} = \frac{\text{Output power}}{\text{Input power}}$$

$$\frac{90}{100} = \frac{12100 \text{ W}}{\text{Input power}}$$

or Input power,

$$\mathcal{E}_1 I_1 = \frac{12100 \times 100}{90} = 13.44 \times 10^3 \text{ W}$$

Current in the primary,

$$I_1 = \frac{\mathcal{E}_1 I_1}{\mathcal{E}_1} = \frac{13.44 \times 10^3}{220} = 61.1 \text{ A}$$

Power loss in the primary

$$= I_1^2 R_1 = (61.1)^2 \times 0.2 = \mathbf{746.61\ W}$$

Power loss in the secondary

$$= I_2^2 R_2 = (11)^2 \times 2.0 = \mathbf{242\ W}.$$

Problems For Practice

1. A transformer has 300 primary turns and 2400 secondary turns. If the primary supply voltage is 230 V, what is the secondary voltage? **[NCERT]**
(Ans. 1.84 kV)
2. A transformer has 200 primary turns and 150 secondary turns. If the operating voltage for the load connected to the secondary is measured to be 300 V, what is the voltage supplied to the primary? **[NCERT]**
(Ans. 400 V)
3. The ratio of the number of turns in the primary and the secondary coils of a step up transformer is 1 : 200. It is connected to a.c. mains of 200 V.

Calculate the voltage developed in the secondary. Determine the value of maximum current in the secondary, when a current of 2.0 A flows through the primary. (Ans. 40,000 V, 0.01 A)

4. A transformer of 100% efficiency has 500 turns in the primary and 10,000 turns in the secondary coil. If the primary is connected to 220 V supply, what is the voltage across the secondary coil ? **[ISCE 94]** (Ans. 4400 V)

5. When a voltage of 120 V is impressed across the primary of a transformer, the current in the primary is 1.85 A. Find the voltage across the secondary, when it delivers 150 mA. The transformer has an efficiency of 95%. (Ans. 1406 V)

6. The primary of a transformer has 200 turns and the secondary has 1000 turns. If the power output from the secondary at 1000 V is 9 kW, calculate (*i*) the primary voltage and (*ii*) the heat loss in the primary coil if the resistance of primary is 0.2 Ω and the efficiency of the transformer is 90%. (Ans. 200 V, 500 W)

7. A town situated 20 km away from a power plant generating power at 440 V, requires 6000 kW of electric power at 200 V. The resistance of the two wire line carrying power is 0.4 Ω per km. The town gets power from the line through a 3000 – 220 V step down transformer at a substation in the town.
   (*i*) Find the line power losses in the form of heat.
   (*ii*) How much power must the plant supply, assuming there is negligible power loss due to leakage ? **[CBSE OD 03]**

[Ans. (*i*) 640 kW, (*ii*) 1240 kW]

HINTS

1. $\mathcal{E}_2 = \frac{N_2}{N_1} \cdot \mathcal{E}_1 = \frac{2400}{300} \times 230 = 1840 \text{ V} = \mathbf{1.84\ kV}.$

2. $\mathcal{E}_1 = \frac{N_1}{N_2} \cdot \mathcal{E}_2 = \frac{200}{150} \times 300 = \mathbf{400\ V}.$

3. Here $\frac{N_1}{N_2} = \frac{1}{200}$, $\mathcal{E}_1 = 200$ V, $I_1 = 2.0$ A

$$\mathcal{E}_2 = \frac{N_2}{N_1} \cdot \mathcal{E}_1 = \frac{200}{1} \times 200 = \mathbf{40{,}000\ V}$$

Assuming the transformer to be ideal, we have

$$\mathcal{E}_1 I_1 = \mathcal{E}_2 I_2$$

$$\therefore \quad I_2 = \frac{\mathcal{E}_1 I_1}{\mathcal{E}_2} = \frac{200 \times 2.0}{4000} = \mathbf{0.01\ A}.$$

4. $\mathcal{E}_2 = \frac{N_2}{N_1} \cdot \mathcal{E}_1 = \frac{10{,}000}{500} \times 220 = \mathbf{4400\ V}.$

5. Here $\mathcal{E}_1 = 120$ V, $I_1 = 1.85$ A,

$I_2 = 150 \text{ mA} = 150 \times 10^{-3}$ A, $\eta = 95\% = 0.95$

$$\text{Efficiency, } \eta = \frac{\text{Power output}}{\text{Power input}} = \frac{\mathcal{E}_2 I_2}{\mathcal{E}_1 I_1}$$

$$\mathcal{E}_2 = \frac{\eta \mathcal{E}_1 I_1}{I_2} = \frac{0.95 \times 120 \times 1.85}{150 \times 10^{-3}} = \mathbf{1406\ V.}$$

6. Here $N_1 = 200$, $N_2 = 1000$, $\mathcal{E}_2 = 1000$ V, $R_1 = 0.2\,\Omega$

(*i*) $\mathcal{E}_1 = \frac{N_1}{N_2} \cdot \mathcal{E}_2 = \frac{200}{1000} \times 1000 = \mathbf{200\ V.}$

(*ii*) As efficiency = 90%, therefore

$$\frac{\text{Output power}}{\text{Input power}} \times 100 = 90$$

or

$$\frac{9 \times 10^3}{\text{Input power}} \times 100 = 90$$

$\therefore$ Input power, $\mathcal{E}_1 I_1 = 10{,}000$ W

$$I_1 = \frac{10000}{\mathcal{E}_1} = \frac{10000}{200} = 50 \text{ A}$$

Heat loss in the primary coil

$$= I_1^2 R_1 = (50)^2 \times 0.2 = \mathbf{500\ W.}$$

7. Total length of wire = 20 × 2 = 40 km

Resistance of the wire, $R = 40 \times 0.4 = 16\,\Omega$

$$I_{rms} = \frac{P}{V} = \frac{600 \text{ kW}}{3000 \text{ V}} = 200 \text{ A}$$

(*i*) Power loss in the form of heat $= I_{rms}^2 \times R$

$$= (200)^2 \times 16 = 640000 \text{ W} = \mathbf{640\ kW}.$$

(*ii*) Power supplied by the plant

= Power demand + Power loss

= 600 + 640 = **1240 kW.**

7.25 A.C. GENERATOR

35. *With the help of a labelled diagram, explain the principle, construction and working of an a.c. generator. Derive the expression for the induced emf and current.*

A.C. Generator. *A generator or dynamo is a device which converts mechanical energy into electrical energy.* Actually, the name generator is a misnomer because it does not generate any energy. It just converts mechanical energy into electrical energy.

An ***a.c. generator*** is the one *which produces a current that alternates or changes its direction regularly after a fixed interval of time, i.e., it is a device which converts mechanical energy into alternating form of electrical energy.* The present form of a.c. generator is due to *Nikola Tesla,* the great Yugoslav scientist who built it in the year 1888.

Principle. The working of an a.c. generator is based on the principle of ***electromagnetic induction.*** *When a* closed coil is rotated in a uniform magnetic field with its axis perpendicular to the magnetic field, the magnetic flux linked with the coil changes and an induced emf and hence a current is set up in it.

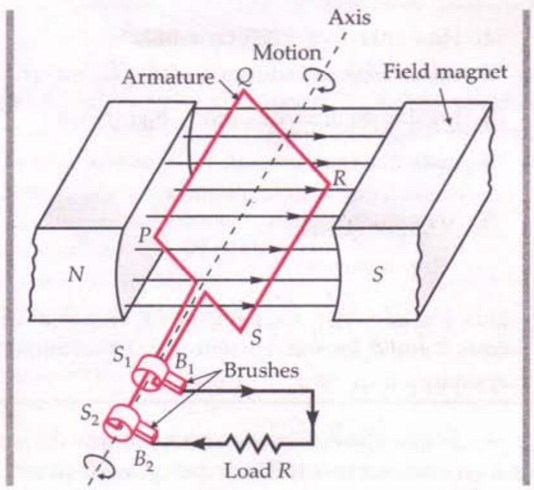


Fig. 7.53 A.C. Generator.

Construction. It essentially consists of the following main parts :

1. ***Field magnet.*** It is a permanent magnet of the horse shoe shape in a small dynamo (magneto) or it is a powerful electromagnet in a large dynamo. It produces a strong magnetic field in the region between its pole-pieces.

2. ***Armature.*** It consists of a rectangular coil *PQRS* having a large number of turns of insulated copper wire wound on a soft iron cylindrical core. The core is laminated one to avoid losses due to eddy currents. The soft iron core concentrates the lines of force to increase the flux density *B*. The armature can be rotated in the magnetic field of the field magnet about an axis perpendicular to field *B*.

3. ***Slip rings.*** The two ends of the armature coil are connected to two coaxial brass rings S_1 and S_2 called slip rings. The rings are rigidly fixed to same shaft which is used to rotate the coil. The slips are insulated from each other as well as from the shaft. As the armature coil rotates, the slip rings also rotate about the same axis of rotation.

4. ***Brushes.*** Two graphite or flexible metallic rods called brushes are lightly pressed against the two slip rings. The brushes B_1 and B_2 remain fixed in their positions and maintain sliding contacts with the rotatable slip rings S_1 and S_2 respectively. It is through these brushes that the current induced in the armature coil is fed to the external circuit by means of line wires.

5. ***Source of energy.*** The armature coil is rotated about its axis with the help of turbine or any other device connected to it. It is the rotational kinetic energy of the turbine which is ultimately converted into electrical energy by the a.c. generator.

Working. As the armature coil rotates, the magnetic flux linked with it changes and so an induced current flows through it. Suppose initially the coil *PQRS* be in the vertical position and it is rotated in the clockwise direction. The side *PQ* moves downward and *SR* moves upward. According to Fleming's right hand rule, the induced current flows from *Q* to *P* and from *S* to *R*. So during the first half rotation of the coil, the induced current flows in the direction *SRQP*, with brush B_1 acting as positive terminal and brush B_2 as negative terminal. During the second half-rotation, the side *PQ* moves upward and *SR* moves downward. The direction of induced current is reversed, *i.e.*, it flows along *PQRS*, so that the brush B_2 now functions as the positive terminal and brush B_1 as the negative terminal. Thus the direction of current in the external circuit is reversed after every half cycle. Hence alternating current is produced by the generator. Such a generator which generates alternating current is called an **a.c. generator** or an **alternator**.

Expression for induced emf. Let

N = number of turns in the coil

A = face area of each turn

B = magnitude of the magnetic field

θ = angle which normal to the coil makes with field $\vec{B}$ at any instant t

ω = the angular velocity with which coil rotates

Then the magnetic flux linked with the coil at any instant t will be

$$\phi = NBA \cos \theta = NBA \cos \omega t$$

By Faraday's flux rule, the induced emf is given by

$$\mathcal{E} = -\frac{d\phi}{dt} = -\frac{d}{dt}(NBA \cos \omega t) = NBA\,\omega \sin \omega t$$

or $\qquad \mathcal{E} = \mathcal{E}_0 \sin \omega t$

where $\mathcal{E}_0 = NBA\,\omega$. When a load of resistance R is connected across the terminals, a current I flows in the external circuit.

$$I = \frac{\mathcal{E}}{R} = \frac{\mathcal{E}_0 \sin \omega t}{R} = I_0 \sin \omega t$$

where $I_0 = \frac{\mathcal{E}_0}{R}$. Both current and voltage vary sinusoidally with time. The power dissipated in the load is supplied by the agent in rotating the coil in the magnetic field.

Hydroelectric power station. In a hydroelectric power station, water is stored in a dam at a height, from where it falls into giant waterwheels or turbines. These turbines are connected to the loops of wires in a.c. generators. The kinetic energy of the falling water thus gets converted into rotational energy of the turbines and ultimately into electrical energy supplied by the generator.

Thermal power station. In a thermal power station, steam is produced by boiling water using coal or oil as fuel. The turbines coupled to the loops of a.c. generators are rotated by steam rushing past them and thus electrical energy is generated.

Nuclear power plant. In a nuclear power plant, a nuclear fuel is used instead of coal to generate electrical energy.

7.26 ADVANTAGES AND DISADVANTAGES OF A.C. OVER D.C.

36. *Write some advantages and disadvantages of a.c. over d.c.*

Advantages of a.c. over d.c.

1. The generation of a.c. is more economical than d.c.
2. The alternating voltage can be easily stepped up or stepped down by using a transformer.
3. The alternating currents can be reduced by using a choke coil without any significant wastage of energy.
4. The alternating currents can be transmitted to distant places without any significant line loss.
5. Also a.c. can be easily converted into d.c. by using rectifiers.
6. A.C. machines are simple and robust and do not require much attention during their use.

Disadvantages of a.c. over d.c.

1. Peak value of a.c. is high $(I_0 = \sqrt{2} I_{rms})$. It is dangerous to work with a.c.
2. In phenomena like electroplating, electro-refining, electrotyping, etc; a.c. cannot be used.
3. A.C. is transmitted more from the surface of conductor than from inside. This is called *skin effect*. Therefore, several fine insulated wires (and not a single thick wire) are required for the transmission of a.c.

Examples based on Generators

Formulae Used

For an a.c. generator,

1. Flux linked, $\phi = NBA \cos \omega t$
2. Instantaneous induced emf, $\mathcal{E} = \mathcal{E}_0 \sin \omega t$
3. Maximum induced emf, $\mathcal{E}_0 = NBA\omega$
4. Instantaneous current, $I = I_0 \sin \omega t$
5. Maximum current, $I_0 = \dfrac{\mathcal{E}_0}{R} = \dfrac{NBA\omega}{R}$.

Units Used

Flux ϕ is in weber, field B in tesla, area A in m^2, emfs $\mathcal{E}$ and $\mathcal{E}_0$ in volt, currents I and I_0 in ampere, resistance R in ohm.

Example 77. *Kamla peddles a stationary bicycle the pedals of which are attached to a 100 turn coil of area 0.10 m^2. The coil rotates at half a revolution per second and it is placed in a uniform magnetic field of 0.01 T perpendicular to the axis of rotation of the coil. What is the maximum voltage generated in the coil ?* [NCERT ; CBSE OD 08]

Solution. Here $N = 100$, $A = 0.10\ \text{m}^2$, $f = 0.5$ Hz, $B = 0.01$ T

The maximum voltage generated in the coil,

$$\mathcal{E}_0 = NBA\omega = NBA \times 2\pi f$$

$$= 100 \times 0.01 \times 0.10 \times 2 \times 3.14 \times 0.5 = \mathbf{0.314\ V}.$$

Example 78. *An a.c. generator consists of a coil of 50 turns and area 2.5 m^2 rotating at an angular speed of 60 rad s^{-1} in a uniform magnetic field $B = 0.30$ T between two fixed pole pieces. The resistance of the circuit including that of the coil is 500 Ω.*

(a) *What is the maximum current drawn from the generator ?*

(b) *What is the flux through the coil when the current is zero ? What is the flux when the current is maximum ?*

(c) *Would the generator work if the coil were stationary and instead the pole pieces rotated together with the same speed as above ?* [CBSE OD 98C ; F 03]

Solution. Here $N = 50$, $A = 2.5\ \text{m}^2$, $\omega = 60$ rad s^{-1}, $B = 0.30\ T$, $R = 500\ \Omega$

Maximum current,

$$I_0 = \frac{\mathcal{E}_0}{R} = \frac{NBA\omega}{R}$$

$$= \frac{50 \times 0.30 \times 2.5 \times 60}{500} = \mathbf{4.5\ A}.$$

(*b*) Current, $I = I_0 \sin \omega t = \frac{NBA\,\omega}{R} \sin \omega t$

Flux, $\phi_B = NBA \cos \omega t$

Current is zero if $\sin \omega t = 0$, or $\omega t = 0°$. Then flux is maximum and its value is

$$\phi_B = NBA \cos 0°$$
$$= NBA = 50 \times 0.30 \times 2.5 \text{ Wb}$$
$$= \mathbf{37.5\ Wb}$$

Current is maximum when $\sin \omega t = 1$ or $\omega t = 90°$. Then flux is zero because

$$\phi_B = NBA \cos 90° = 0.$$

(*c*) Yes, the generator would work if the coil were stationary and the pole pieces are rotated together with the same speed because this will also bring about the necessary flux change.

Example 79. *An a.c. generator consists of a coil of 100 turns and cross-sectional area of 3 m², rotating at a constant angular speed of 60 radians / sec in a uniform magnetic field of 0.04 T. The resistance of the coil is 500 ohm. Calculate (i) maximum current drawn from the generator and (ii) maximum power dissipation in the coil.* [CBSE D 02]

Solution. Here $N = 100$, $A = 3 \text{ m}^2$, $\omega = 60 \text{ rad s}^{-1}$, $B = 0.04 \text{ T}$, $R = 500\,\Omega$

(*i*) Maximum current drawn,

$$I_0 = \frac{\mathcal{E}_0}{R} = \frac{NBA\omega}{R}$$
$$= \frac{100 \times 0.04 \times 3 \times 60}{500} = \mathbf{1.44\ A}$$

(*ii*) Max. power dissipation

$$= \mathcal{E}_{eff} \cdot I_{eff} = \frac{\mathcal{E}_0}{\sqrt{2}} \cdot \frac{I_0}{\sqrt{2}} = \frac{I_0^2 R}{2}$$
$$= \frac{(1.44)^2 \times 500}{2} = \mathbf{518.4\ W.}$$

Example 80. *A generator develops an emf of 120 V and has a terminal potential difference of 115 V, when the armature current is 25 A. What is the resistance of the armature ?*

Solution. Here $\mathcal{E} = 120$ V, $V = 115$ V, $I = 25$ A

As $$I = \frac{\mathcal{E} - V}{R}$$

$$\therefore \quad R = \frac{\mathcal{E} - V}{I} = \frac{120 - 115}{25} = \mathbf{0.2\,\Omega.}$$

Problems For Practice

1. An armature coil consists of 20 turns of wire, each of area $A = 0.09 \text{ m}^2$ and total resistance $15.0\,\Omega$. It rotates in a magnetic field of 0.5 T at a constant frequency of $\frac{150}{\pi}$ Hz. Calculate the value of (*i*) maximum (*ii*) average induced emf produced in the coil. **[CBSE Sample Paper 08]**
[Ans. (*i*) 270 V (*ii*) 0]

2. An a.c. generator consists of a coil of 50 turns and area 2.5 m^2 rotating at an angular speed of 60 rad s^{-1} in a uniform magnetic field of 0.30 T. The resistance of the circuit including that of the coil is $500\,\Omega$. (*i*) Find the peak value of current drawn from the generator. (*ii*) What is the flux through the coil when the current is zero ? (Ans. 45 A, 37.5 Wb)

3. An a.c. generator consists of a coil of 2000 turns each of area 80 cm^2 and rotating at an angular speed of 200 rpm in a uniform magnetic field of 4.8×10^{-2} T. Calculate the peak and rms values of emf induced in the coil. **[Punjab 02]**
(Ans. 16.085 V, 11.375 V)

HINTS

1. (*i*) $\mathcal{E}_0 = NBA\omega = NBA \times 2\pi f$
$$= 20 \times 0.5 \times 0.09 \times 2\pi \times \frac{150}{\pi} \text{ V}$$
$$= \mathbf{270\ V.}$$
(*ii*) $\mathcal{E}_{av} = \mathbf{0.}$

2. (*i*) Here $N = 50$, $A = 2.5 \text{ m}^2$, $\omega = 60 \text{ rad s}^{-1}$,
$B = 0.30 \text{ T}$, $R = 500\,\Omega$
Peak value of current,
$$I_0 = \frac{\mathcal{E}_0}{R} = \frac{NBA\omega}{R}$$
$$= \frac{50 \times 0.30 \times 2.5 \times 60}{500} = \mathbf{4.5\ A.}$$
(*ii*) When current is zero, induced *emf* = 0. As the induced emf is the rate of change of magnetic flux, so magnetic flux must be maximum.
$\therefore \quad \phi = NBA = 50 \times 0.30 \times 25 = \mathbf{37.5\ Wb.}$

3. Here $N = 2000$, $A = 80 \text{ cm}^2 = 80 \times 10^{-4} \text{ m}^2$,
$$B = 4.8 \times 10^{-2} \text{T}, \quad f = 200 \text{ rpm} = \frac{200}{60} \text{ rps},$$
$$\omega = 2\pi f = \frac{2\pi \times 200}{60} = \frac{20\pi}{3} \text{ rad s}^{-1}$$
$$\mathcal{E}_0 = NBA\omega$$
$$= 2000 \times 4.8 \times 10^{-2} \times 80 \times 10^{-4} \times \frac{2\pi}{3}$$
$$= \mathbf{16.085\ V}$$
$$\mathcal{E}_{rms} = 0.707\,\mathcal{E}_0 = 0.707 \times 16.085$$
$$= \mathbf{11.375\ V.}$$

VERY SHORT ANSWER CONCEPTUAL PROBLEMS

Problem 1. What is the average value of a.c. over a cycle and why ?

Solution. Zero, because a.c is positive during one half cycle and equally negative during other half cycle.

Problem 2. When an alternating current is passed through a moving coil galvanometer, it shows no deflection. Why ?

Solution. A moving coil galvanometer measures average value of current, which is zero for a.c. over every cycle. So galvanometer shows no deflection.

Problem 3. Why a d.c. voltmeter and d.c. ammeter cannot read a.c. ? [Punjab 98C]

Solution. The average value of a.c. over a cycle is zero. So a d.c. voltmeter/ammeter will show zero reading with alternating voltage/current.

Problem 4. On which effect of current, a.c. ammeters are based ? Give reason.

Solution. A.C. ammeters are based on heating effect of current because this effect does not depend on the direction of current.

Problem 5. Can we define rms value of a.c. in terms of the chemical effect of current ?

Solution. No, because the chemical effects are reversed when the direction of current is reversed.

Problem 6. Which value of current do you read with an a.c. ammeter ?

Solution. Root mean square value of the current.

Problem 7. Can we use a.c. of frequency 15 cps for lighting purpose ?

Solution. Yes, because the fluctuations in current will be so rapid (30 times per second) that the bulb will appear glowing continuously due to persistence of vision.

Problem 8. A 110 V d.c. heater is used on an a.c. source, such that the heat produced is the same. What would be the rms value of the alternating voltage ? [ISCE 96]

Solution. By definition, rms value of the alternating voltage = **110 V.**

Problem 9. 220 V a.c. is more dangerous than 220 V d.c. Why ?

Solution. 220 V a.c. has a peak voltage of $220 \times \sqrt{2}$ = 311 V, while 220 V d.c. has a peak voltage of 220 V only, so a.c. of same voltage is more dangerous than d.c.

Problem 10. Find the time required for a 60 Hz alternating current to reach its peak value starting from zero.

Solution. Time period of a.c.,

$$T = \frac{1}{f} = \frac{1}{60}\ \text{s}$$

The current will take one-fourth of the time period to reach its peak value starting from zero.

$\therefore$ Required time, $t = \frac{T}{4} = \frac{1}{4 \times 60} = \mathbf{\frac{1}{240}\ s.}$

Problem 11. What is meant by the statement that the current through an inductor lags behind the emf across it by $\pi / 2$?

Solution. This means that in an inductive a.c. circuit whichever value emf attains, current attains a similar value a quarter of cycle later. For example, if emf attains its maximum value at $t = 0$, then current attains its maximum value at $t = T/4$, and so on.

Problem 12. The frequency of a.c. is doubled. How do R, X_L and X_C get affected ?

Solution. (*i*) R remains unaffected.

(*ii*) X_L gets doubled, because $X_L \propto f$.

(*iii*) X_C becomes one-half of the original value, because $X_C \propto \frac{1}{f}$.

Problem 13. What is the reactance of an inductor in a d.c. circuit ? [Punjab 98C]

Solution. For d.c., $f = 0$, therefore,

inductive reactance $X_L = 2\pi f L = 0.$

Problem 14. Capacitors block d.c. Why ? [Punjab 95]

Solution. For d.c., $f = 0$, therefore,

$$X_C = \frac{1}{2\pi f C} = \infty$$

So a capacitor does not allow d.c. to flow through it *i.e.*, it blocks d.c.

Problem 15. For very high frequency a.c. supply, a capacitor behaves like a pure conductor. Why ?

Solution. As $X_C = \frac{1}{2\pi f C}$

$$\therefore \quad X_C \propto \frac{1}{f}$$

Hence at very high frequency, capacitive reactance becomes negligibly small and capacitor behaves like a pure conductor.

Problem 16. Show that an inductor offers an easy path to d.c. and a resistive path to a.c.

Solution. Inductive reactance,

$$X_L = 2\pi f L$$

Clearly, X_L is zero for d.c. ($f = 0$) and has a finite value for a.c. (finite f). Hence an inductor offers an easy path to d.c. and a resistive path to a.c.

Problem 17. An ideal inductor is in turn put across 220 V, 50 Hz and 220 V, 100 Hz supplies. Will the current flowing through it in the two cases be the same or different ? [CBSE OD 98]

Solution. The current flowing through the inductor will be more in first case because inductive reactance ($X_L = 2\pi f L$) is less than that in second case.

Problem 18. A choke coil and a bulb are connected in series to an a.c. source. The bulb shines brightly. How does its brightness change when an iron core is inserted in the choke coil ? [CBSE OD 95]

Solution. When the iron core is inserted in the choke coil, the self-inductance L increases. Consequently, the inductive reactance, $X_L = \omega L$ increases. This decreases the current in the circuit and the bulb glows dimmer.

Problem 19. Voltages across L and C in series are 180° out of phase. Comment.

Solution. Given a current in series LC, voltage in L leads current by 90° phase and voltage in C lags behind current by 90° phase. So voltages in L and C differ by a phase of 180°.

Problem 20. When L and C are connected in parallel, currents in L and C are 180° out of phase. Comment.

Solution. Given an applied voltage across parallel LC, current in L lags behind voltage by 90° phase, current in C leads voltage by 90° phase. So currents in L and C are 180° out of phase.

Problem 21. When are the voltage and current in *LCR*-series a.c. circuit in phase ? [Haryana 02]

Solution. When $X_L = X_C$, voltage and current in a series *LCR*-circuit are in same phase.

Problem 22. If the frequency of the a.c. source in a series *LCR*-circuit is increased, how does the current in the circuit change ? [CBSE D 98C]

Solution. With the increase in frequency, the current in a series *LCR*-circuit first increases, attains a maximum value (at $f = f_r$) and then decreases.

Problem 23. The hot wire ammeter in Fig. 7.54(*a*) shows some deflection but not in Fig. 7.54(*b*). Why ?

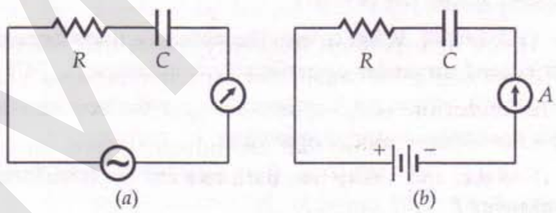


Fig. 7.54

Solution. Capacitor in Fig. 7.54(*a*) conducts a.c. and so ammeter shows some deflection. Capacitor in Fig. 7.54(*b*) blocks d.c. and so ammeter shows no deflection.

Problem 24. When a series *LCR*-circuit is brought into resonance, the current in the circuit increases to a large value. Why ?

Solution. In resonance condition, the impedance of the *LCR*-circuit becomes minimum and so current in the circuit rises to a maximum value.

Problem 25. An air core coil and an electric bulb are connected in series across a 220 V, 50 Hz a.c. source. The bulb glows with some brightness. How will the glow of the bulb be affected on introducing a capacitor in series in circuit ? Justify your answer. [CBSE OD 94]

Solution. Impedance before inserting capacitor,

$$Z_1 = \sqrt{R^2 + \omega^2 L^2}$$

Impedance after inserting capacitor,

$$Z_2 = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}.$$

Clearly, $Z_2 < Z_1$. So when capacitor is inserted, current through the circuit increases and the bulb glows more brightly.

Problem 26. What is the maximum value of power factor ? When does it occur ?

Solution. ***One.*** For a purely resistive circuit, $\phi = 0$.

$\therefore$ Power factor, $\cos\phi = \cos 0 = 1$.

Problem 27. What is the minimum value of power factor ? When does it occur ?

Solution. ***Zero.*** For a purely inductive or capacitive circuit, $\phi = \pm\pi/2$.

$\therefore$ Power factor, $\cos\phi = \cos(\pm\pi/2) = 0$.

Problem 28. Why power correction is must in heavy machinery ? [Punjab 01, 04]

Solution. $P_{av} = V_{rms} I_{rms} \cos\phi$.

A heavy machinery needs large power. For a given supply voltage, it requires either large current or improvement of power factor. For supplying large current, thick wires have to be used which is not economical. In practice, power factor ($\cos\phi$) is increased by decreasing ϕ. This is done by using a capacitor of appropriate capacitance.

Problem 29. In an a.c. circuit, there is no power consumption in an ideal inductor. Why ? [Punjab 98C]

Solution. In an a.c. circuit containing inductor only, the voltage leads the current by phase angle $\pi/2$, so average power consumed per cycle is zero.

$$P_{av} = V_{rms} \cdot I_{rms} \cdot \cos\frac{\pi}{2} = 0$$

Problem 30. A capacitor of capacitance C is connected across an a.c. source of maximum emf V_0. What is the average power dissipated in the circuit ? [CBSE D 93C]

Solution. In an a.c. circuit containing capacitor only, emf lags behind the current by phase angle $\pi/2$, so average power consumed per cycle is zero.

$$P_{av} = V_{rms} \cdot I_{rms} \cos\left(-\frac{\pi}{2}\right) = 0.$$

Problem 31. **A perfect self-inductance when connected to an a.c. source does not produce any heating effect, yet reduces current in the circuit. Why ?**

Solution. As the average power consumed per cycle in an inductive a.c. circuit is zero, so no heating is caused. But the inductive reactance, $X_L = \omega L = 2\pi f L$ plays the same role in a.c. circuit as the resistance in d.c. circuit. So an inductance reduces current in an a.c. circuit.

Problem 32. **When is the current in a.c. circuit wattless ?**

Solution. The current in an a.c. circuit is wattless when the phase difference between current and voltage is $\pi/2$. It can be obtained by using an inductor or a capacitor in the circuit.

Problem 33. **Which is the best method of reducing current in an a.c. circuit and why ?** [CBSE D 96C]

Solution. The current in an a.c. circuit can be best reduced by using a choke coil or capacitor. There is no dissipation of energy in these devices.

Problem 34. **Can we use a capacitor instead of a choke coil for reducing current in an a.c. circuit ? Give reason.** [CBSE OD 96C]

Solution. Yes, because the average power dissipated per cycle in an ideal capacitor is also zero.

Problem 35. **With reference to alternating currents and voltages, state any one fundamental difference between resistance and reactance.** [ISCE 98]

Solution. The resistance of any component is independent of the frequency of a.c. while the reactance changes with the change in frequency of the a.c. source.

Problem 36. **What do you mean by charging and discharging of a capacitor ?**

Solution. The process of storing charge on the plates of a capacitor is called charging, and the process of withdrawing charge from the plates is called discharging of the capacitor.

Problem 37. **On what factors does the rate of charging and discharging of a capacitor depend ?**

Solution. The rate of charging and discharging of a capacitor depends on (*i*) its capacitance *C* and (*ii*) the resistance *R* of the circuit.

Problem 38. **Show graphically the variation of charge Q with time, when a condenser is charged.**

[CBSE OD 90C]

Solution. Fig. 7.55 shows the variation of charge *Q* with time *t*, when a capacitor is charged.

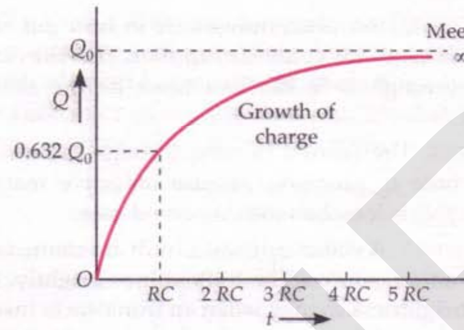


Fig. 7.55

Problem 39. **A capacitor is connected in series to an ammeter across a d.c. source. Why does the ammeter show a momentary deflection during the charging of he capacitor ? What would be the deflection when it is fully charged ?** [CBSE OD 14C]

Solution. The momentary deflection is due to the transient current flowing through the circuit when the capacitor is getting charged.

The deflection would be zero when the capacitor gets fully charged.

Problem 40. **Define time constant of *RC*-circuit.**

Solution. The time constant of *RC*-circuit may be defined as the time in which the capacitor gets charged to 0.632 times the maximum charge Q_0.

Problem 41. **Why can't transformer be used to step up d.c. voltage ?** [CBSE OD 11 ; Haryana 01]

Solution. The d.c. supply does not produce a changing magnetic flux in the primary and hence no emf is set up in the secondary of the transformer.

Problem 42. **Does a step up transformer contradict the principle of conservation of energy ?** [Haryana 93, 94]

Solution. No. In a transformer, energy is neither created nor destroyed.

Problem 43. **Does a transformer change the frequency of a.c. ?**

Solution. No. The frequency of alternating voltage obtained across the secondary is same as that of voltage applied across the primary.

Problem 44. **What causes the core of a transformer to get heated up under operation ?** [ISCE 95]

Solution. The eddy currents set up in the iron core heat up a transformer under operation.

Problem 45. **Why is the core of a transformer laminated ?**

Solution. The core of a transformer is laminated so as to minimise energy losses due to eddy currents.

Problem 46. **Why is the core of a transformer made of a magnetic material of high permeability ?**

[CBSE Sample Paper 1990]

Solution. Due to high permeability of core material, the magnetic lines of force will crowd the core. Most of the flux linked with the primary will pass through the secondary. Flux leakage decreases and efficiency of transformer increases.

Problem 47. **A transformer is used to step down a.c. voltage. Which appliance will you use to step down d.c. voltage ?**

Solution. A pure resistor can be used to step down d.c. voltage.

Problem 48. **The core of transformer is made of a material having narrow hysteresis loop. Why ?**

Solution. The alternating current carries the iron core through the cycles of magnetisation and demagnetisation. Work is done in each of these cycles and is lost as heat. This is called hysteresis loss which can be reduced by using core material having narrow hysteresis loop.

Problem 49. **Why is choke preferred to rheostat in controlling a.c. supply ?** [Himachal 98C ; ISCE 93]

Solution. A choke reduces current in an a.c. circuit without dissipating any power. A rheostat also reduces current but it dissipates energy in the form of heat.

Problem 50. **Is there any device by which direct current can be controlled without any loss of energy ? Can a choke coil be used ?**

Solution. No, there is no device which can control d.c. without any loss of energy. A choke coil cannot control d.c.

Problem 51. **Why a choke coil cannot be used in d.c. ?** [Haryana 01]

Solution. The reactance of a choke coil is given by

$$X_L = 2\pi f L$$

For d.c., $f = 0$, so $X_L = 0$.

Thus a choke coil does not oppose current in a d.c. circuit. So it cannot be used to control d.c.

Problem 52. **In India, domestic power supply is at 220 V, 50 Hz, while in USA it is 110 V, 50 Hz. Give one advantage and one disadvantage of 220 V supply over 110 V supply.** [CBSE OD 04]

Solution. ***Advantage.*** The power loss at 220 V supply is less than that at 110 V.

Disadvantage. The 220 V is more dangerous because its peak value (311 V) is much higher than the peak value (155.5 V) for 110 V supply.

Problem 53. **A solenoid with an iron core and a bulb are connected to a d.c. source. How does the brightness of the bulb change, when the iron core is removed from the solenoid ?** [CBSE OD 04]

Solution. The brightness of the bulb remains unchanged, because the solenoid does not offer any reactance ($X_L = 2\pi f L$) in d.c. circuit ($f = 0$).

Problem 54. **Fig. 7.56 shows the variation of an alternating emf with time. What is the average value of the emf for the shaded part of the graph ?** [CBSE Sample Paper 04]

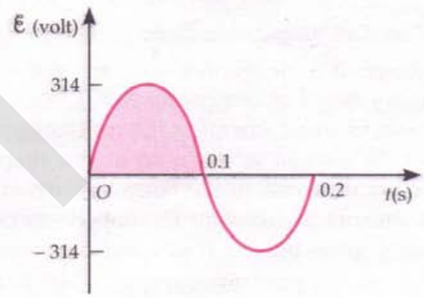


Fig. 7.56

Solution. $\mathcal{E}_{av} = \frac{2}{\pi} \mathcal{E}_0$

$$= \frac{2}{3.14} \times 314 = \mathbf{200\ V}.$$

SHORT ANSWER CONCEPTUAL PROBLEMS

Problem 1. **Answer the following questions :**

(*a*) **For circuits used for transporting electric power, a low power factor implies large power loss in transmission. Explain.** [CBSE OD 15]

(*b*) **Power factor can often be improved by the use of a capacitor of appropriate capacitance in the circuit. Explain.**

(*c*) **A lamp is connected in series with a capacitor. Predict your observations for d.c. and a.c. connections. What happens in each case if the capacitance of the capacitor is reduced ?** [NCERT ; CBSE D 13C]

Solution. (*a*) As $P_{av} = V_{eff} \cdot I_{eff} \cdot \cos\phi$

$$\therefore \quad I_{eff} = \frac{P_{av}}{V_{eff} \cos\phi}$$

To supply a given power, low power factor ($\cos\phi$) requires a larger current to be supplied. This results in larger ($I^2 R$) heat losses.

(*b*) Power factor, $\cos\phi = \frac{R}{Z} = \frac{R}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$

By using a capacitor of suitable capacitance, impedance Z can be made small and hence power factor large. In the limiting case when Z approaches R, the power factor becomes unity (maximum).

(c) For d.c., capacitor is an open circuit because $X_C = 1/\omega C = \infty$. The lamp will not glow at all, even if C is reduced. For a.c., the lamp will glow because capacitor conducts a.c. If C is reduced, the reactance X_C will increase and the brightness of the lamp will decrease further.

Problem 2. **Distinguish between resistance, reactance and impedance of an a.c. circuit.**

[Punjab 02 ; CBSE D 2000]

Solution. **Resistance.** *It is the opposition offered by a pure resistor to the flow of current in a circuit.* It depends on the nature of material of the conductor. It does not depend on the frequency of a.c. Its SI unit is ohm.

Reactance. *The non-resistive opposition to the flow of current is called reactance.* It may be an inductive or a capacitive reactance. It is equal to the ratio of effective p.d. across the inductor (or the capacitor) to the current flowing through it.

Inductive reactance, $X_L = \omega L = 2\pi f L,\ X_L \propto f$

Capacitive reactance,

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C},\ X_C \propto \frac{1}{f}$$

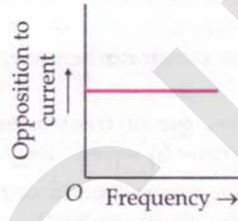
The SI unit of reactance is ohm.

Impedance. *It is the effective resistance of an a.c. circuit containing any two or all three elements R, L and C.* It plays the same role in an a.c. circuit as the resistance plays in a d.c. circuit. It is equal to the ratio of the effective p.d. across the entire circuit to the effective current flowing through it. Its unit is also ohm. The impedance of a series LCR-circuit is given by

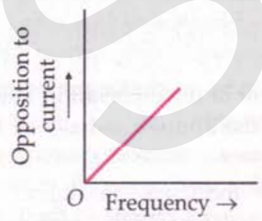
$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

Problem 3. **The graphs shown in Figs. 7.57 (a) & (b) represent the variation of opposition offered by the circuit element to the flow of alternating current, with the frequency of the applied emf. Identify the circuit element corresponding to each graph.**

[CBSE Sample Paper 04]



(a)



(b)

Fig. 7.57

Solution. (i) Graph 7.57(a) is for a pure resistor, because the opposition to current is independent of frequency.

(ii) Graph 7.57(b) is for a pure inductor, because the opposition to current is directly proportional to the frequency of applied emf.

Problem 4. **Compare the important features of resistance, reactance and impedance for an a.c. circuit.**

Solution.

Resistance (R)	Reactance (X_L or X_C)	Impedance (Z)
1. It is opposition to the flow of current by a pure resistor.	It is resistance offered by a pure inductor or pure capacitor to the flow of a.c.	It is effective resistance offered by LR- or CR- or LCR-circuit to the flow of a.c.
2. $R = \frac{V}{I}$	X_L or $X_C = \frac{V_{eff}}{I_{eff}}$	$Z = \frac{V_{eff}}{I_{eff}}$
3. R does not depend on frequency of a.c.	As f increases, X_L increases while X_C decreases. $X_L = 2\pi f L,\ X_L \propto f$ $X_C = \frac{1}{2\pi f C},\ X_C \propto \frac{1}{f}$	Z depends on reactance $X_L \sim X_C$. $Z = \sqrt{R^2 + (X_L - X_C)^2}$
4. Power dissipation across R is maximum.	Power dissipation is zero across X_L or X_C	Power dissipation depends on power factor, $\cos\phi = \frac{R}{\sqrt{R^2 + (X_L - X_C)^2}}$
5. Current and voltage across R are in same phase.	In X_L, voltage leads by 90°. In X_C, current leads by 90°.	Phase angle between voltage and current depends on $(X_L \sim X_C)$.
6. Unit of R is Ω.	Unit of X_L or X_L is Ω.	Unit of Z is Ω.

Problem 5. Prove that high frequency a.c. can pass through a pure capacitor easily but not through a pure inductor. [Punjab 03, 04]

Solution.

Inductive reactance,

$$X_L = 2\pi\ f\ L \quad i.e., \quad X_L \propto f$$

Capacitive reactance,

$$X_C = \frac{1}{2\pi\ f\ C} \quad i.e., \quad X_C \propto \frac{1}{f}$$

Clearly, for a high frequency a.c., X_C is small and X_L is large. That is why a high frequency a.c. can pass through a pure capacitor easily but not through a pure inductor.

Problem 6. (*i*) Draw the graphs showing variation of inductive reactance and capacitive reactance with frequency of applied a.c. source. (*ii*) Can the voltage drop across the inductor or the capacitor in a series *LCR* circuit be greater than the applied voltage of the a.c. source ? Justify your answer. [CBSE OD 02 ; D 05]

Solution. (*i*) Inductive reactance, $X_L = 2\pi f\ L$ *i.e.*, $X_L \propto f$. As shown in Fig. 7.58(*a*), graph of X_L against f is a straight line with a positive slope. As f increases, X_L also increases.

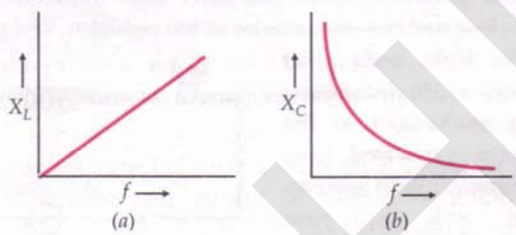


Fig. 7.58

Capacitive reactance, $X_C = \frac{1}{2\pi\ f\ C}$ *i.e.*, $X_C \propto \frac{1}{f}$

Figure 7.58(*b*) shows the variation of X_C with f. As f increases, X_C decreases.

(*ii*) Yes, the voltage drop across the inductor or the capacitor in a series circuit can be greater than the applied voltage. These two voltages are not in same phase, hence they cannot be added like ordinary numbers.

Problem 7. A capacitor blocks d.c. and allows a.c. to flow through it. Explain. [Haryana 95, 01]

Solution. The reactance of a capacitor is given by

$$X_C = \frac{1}{2\pi\ f\ C} \qquad i.e., \qquad X_C \propto \frac{1}{f}$$

Frequency of d.c., $f = 0$, therefore, $X_C = \infty$. That is why a capacitor does not allow d.c. to flow through it *i.e.*, a capacitor blocks d.c. For a.c., frequency f has a finite value and value of X_C is comparatively smaller. Thus a capacitor allows a.c. to flow through it. The conduction of a.c. is due to continuous charging and discharging of the capacitor.

Problem 8. You are given an air coil, a bulb, an iron rod and a source of electricity. Suggest a method to find whether the given source is d.c. or a.c. Explain your answer. [CBSE F 93C, 94]

Solution. The air core coil, bulb and the source of electricity are connected in series.

If the bulb shines brightly, the source must be d.c. type. The insertion of iron rod in the core coil will not affect the bulb's brightness. This is because for a d.c. source ($f = 0$), inductive reactance

$$X_L = 2\pi\ f\ L = 0$$

If the bulb shines dimly, the source is a.c. type because now the coil offers reactance X_L. The bulb will dim further when the iron rod is inserted in the coil which increases the coil's reactance.

Problem 9. Figures 7.59(*a*), (*b*) and (*c*) show three a.c. circuits in which equal currents are flowing. If the frequency of emf be increased, how will the current be affected in these circuits ? Give reason for your answer. [CBSE OD 04C ; D 11C]

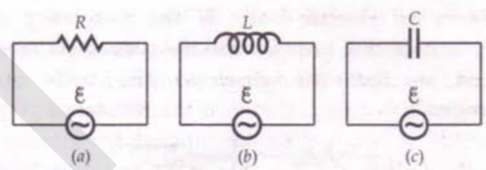


Fig. 7.59

Solution. (*a*) R is not affected by frequency. So current does not change on increasing f.

(*b*) Inductive reactance, $X_L = 2\pi\ f\ L$. When the frequency f is increased, X_L increases and hence current in the circuit decreases.

(*c*) Capacitive reactance, $X_C = \frac{1}{2\pi\ f\ C}$. As the frequency f is increased, X_C decreases and hence current in the circuit increases.

Problem 10. Does the current in an a.c. circuit lag, lead or remain in phase with the voltage of frequency applied to the circuit, when (*i*) $f = f_r$ (*ii*) $f < f_r$ and (*iii*) $f > f_r$, where f_r is the resonant frequency ? [CBSE OD 06]

Solution. (*i*) $f = f_r$ occurs when $X_L = X_C$. Then the circuit becomes purely resistive. So current and voltage will be in the same phase.

(*ii*) $X_L = 2\pi\ f\ L$ and $X_C = \frac{1}{2\pi\ f\ L}$

When $f < f_r$, X_L is small and X_C is large.

The circuit is capacitive. So current leads the voltage in phase.

(*iii*) When $f > f_r$, X_L is large and X_C is small.

The circuit is inductive. So current lags behind the voltage in phase.

Problem 11. When a.c. circuit with L, C and R in series is brought into resonance, the current has large value. Why ? If the capacitance C is increased, will current increase or decrease ? Explain with suitable relation. [CBSE Sample Paper 1990]

Solution. The impedance of a series LCR-circuit is given by

$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

When the circuit is brought into resonance,

$$\omega L = \frac{1}{\omega C} \quad \text{and} \quad Z = R$$

Thus the impedance Z of the circuit is minimum at resonance and hence the current through the circuit is maximum.

If capacitance C is increased, the reactance $1/\omega C$ decreases and so impedance Z increases. Consequently, the current decreases.

Problem 12. In the circuit shown in Fig. 7.60, R represents an electric bulb. If the frequency of the supply is doubled, how should the values of C and L be changed so that the glow in the bulb remains unchanged ? [CBSE Sample Paper 08]

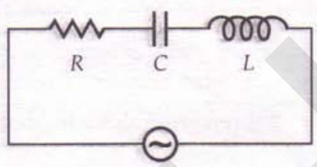


Fig. 7.60

Solution. Current in the LCR-circuit is given by

$$I_{eff} = \frac{\mathcal{E}_{eff}}{\sqrt{R^2 + \left(2\pi f L - \frac{1}{2\pi f C}\right)^2}}$$

When the frequency f of the supply is doubled, both the values of L and C should be halved, so that the reactance $\left(2\pi f L - \frac{1}{2\pi f C}\right)$ remains unchanged and hence current in the circuit remains the same. Then the glow of the bulbs will remain unchanged.

Problem 13. An electric heater is connected, turn by turn, to a d.c. and a.c. sources of equal voltages. Will the rate of heat production be same in the two cases ? Explain. [CBSE Sample Paper 11]

Solution. The element of the heater is a coil, having inductance L and resistance R. Hence, for a.c., its effective resistance $\left(Z = \sqrt{R^2 + \omega^2 L^2}\right)$ will be greater than its pure resistance R for d.c. Consequently, for the same voltage, the rate of heat production will be less for a.c. than that for d.c.

Problem 14. An inductor 'L' of reactance X_L, is connected in series with a bulb 'B' to an a.c. source as shown in Fig. 7.61.

Briefly explain how does the brightness of the bulb change, when (*i*) number of turns of the inductor is reduced and (*ii*) a capacitor of reactance $X_C = X_L$ is included in series in the same circuit. [CBSE D 02, 15]

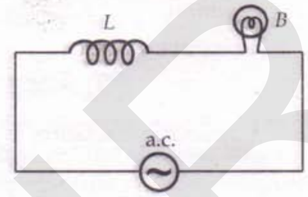


Fig. 7.61

Solution. (*i*) When the number of turns in the inductor is reduced, its reactance X_L decreases. The current in the circuit increases and hence brightness of the bulb increases.

(*ii*) With capacitor of reactance $X_C = X_L$, the impedance $Z = \sqrt{R^2 + (X_L^2 - X_C)^2} = R$ becomes minimum. The current in the circuit becomes maximum. The bulb glows with maximum brightness.

Problem 15. A light bulb and an open coil inductor are connected to an a.c. source through a key as shown in Fig. 7.62.

The switch is closed and after some time, an iron rod is inserted into the interior of the inductor. The glow of the light bulb : (*a*) increases ; (*b*) decreases ; (*c*) is unchanged, as the iron rod is inserted. Give your answer with reasons. What will be your answer if ac source is replaced by d.c. ? [CBSE D 15 ; NCERT]

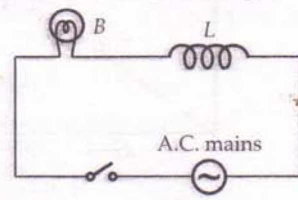


Fig. 7.62

Solution. When the iron rod is inserted in the coil, its inductance L increases μ_r times. Consequently, its reactance, $X_L = 2\pi f L$ increases.

Hence the impedance $Z = \sqrt{R^2 + X_L^2}$ of the circuit increases. This decreases the current and also the glow of the bulb decreases.

When the a.c. source is replaced by a d.c. source, the glow of the bulb increases and the insertion of iron rod into the inductor does not affect the glow of the bulb. This is because for d.c., $f = 0$ and $X_L = 2\pi f L = 0$.

Problem 16. An electric lamp having coil of negligible inductance is connected in series with a capacitor and an a.c. source. How does the brightness of the lamp change on reducing the (*i*) capacitance, and (*ii*) the frequency ? Justify your answer.

Solution. Capacitive reactance, $X_C = \frac{1}{2\pi f C}$

(*i*) When capacitance C is reduced, X_C increases. The current through the circuit decreases. Hence the brightness of the bulb decreases.

(*ii*) When frequency f of a.c. decreases, X_C increases. The current decreases and hence brightness also decreases.

Problem 17. **A capacitor 'C', a variable resistor 'R' and a bulb 'B' are connected in series to the a.c. mains in circuit as shown in Fig. 7.63. The bulb glows with some brightness. How will the glow of the bulb change if (*i*) a dielectric slab is introduced between the plates of the capacitor, keeping resis- tance R to be the same ; (*ii*) the resistance R is increased keeping the same capacitance ?** [CBSE D 14]

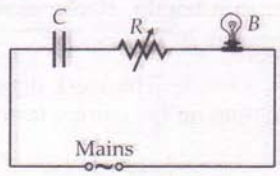


Fig. 7.63

Solution. (*i*) When the dielectric slab is introduced, the capacitance C increases. This decreases the capacitive reactance $(X_C = 1/\omega C)$ and increases the current in the circuit. Therefore, the bulb glows with more brightness.

(*ii*) When the resistance is increased, the current in the circuit decreases. This decreases the glow of the bulb.

Problem 18. **The current flowing through an inductor of self inductance L is continuously increasing. Plot a graph showing the variation of :**

(*i*) Magnetic flux versus the current

(*ii*) Induced emf versus dI/dt

(*iii*) Magnetic potential energy stored versus the current. [CBSE D 14]

Solution. (*i*) As $\phi \propto I$, so the graph of ϕ versus I is a straight line as shown in Fig. 7.64(*a*)

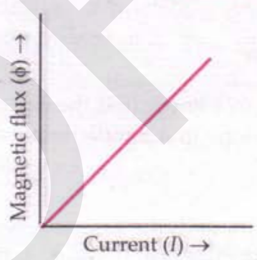
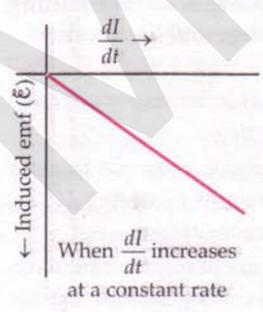


Fig. 7.64 (*a*)

(*ii*) As $\mathcal{E} = -L\frac{dI}{dt}$, the graph of $\mathcal{E}$ versus $\frac{dI}{dt}$ is a straight line with $\mathcal{E}$ on the –ve side.



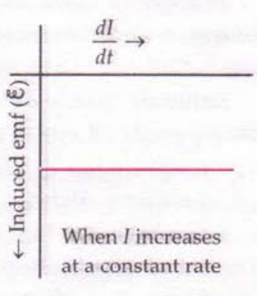


Fig. 7.64 (*b*) (*c*)

(*iii*) Magnetic energy stored,

$$U = \frac{1}{2}LI^2$$

$$\Rightarrow \quad U \propto I^2$$

So the graph of U versus I is a parabola as shown in Fig. 7.64(*d*).

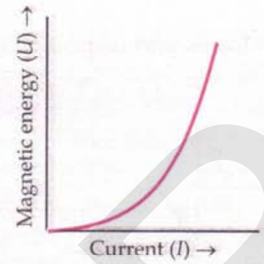


Fig. 7.64 (*d*)

Problem 19. **Show graphically the variation of instantaneous power P with angle ωt when alternating voltage $V = V_0 \sin \omega t$ is applied across (*i*) a pure resistor (*ii*) a pure inductor and (*iii*) a pure capacitor.**

Solution. (*i*) Instantaneous voltage and current are in same phase in a pure resistor.

$$V = V_0 \sin \omega t \quad \text{and} \quad I = I_0 \sin \omega t$$

Instantaneous power, $P = VI = V_0 I_0 \sin^2 \omega t$

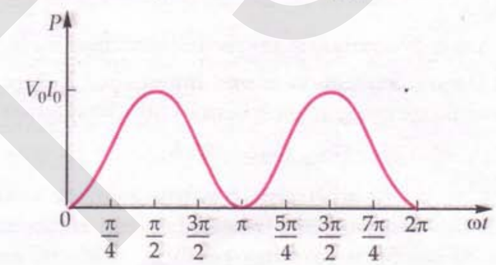


Fig. 7.65 (*a*) Variation of instantaneous power with ωt in a resistive a.c. circuit.

Instantaneous power is +ve at every instant except when both V and I are zero.

(*ii*) In a pure inductor, current lags behind voltage by a phase angle of $\pi/2$

$$V = V_0 \sin \omega t \text{ and } I = I_0 \sin(\omega t - \pi/2) = -I_0 \cos \omega t$$

Instantaneous power,

$$P = VI = -V_0 I_0 \sin \omega t \cos \omega t = -\frac{V_0 I_0}{2} \sin 2\omega t$$

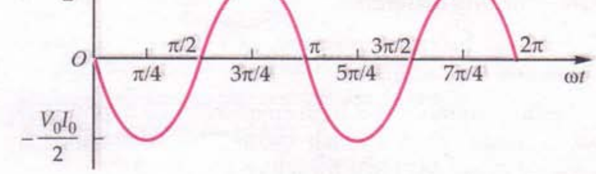


Fig. 7.65 (*b*) Variation of instantaneous power with ωt in an inductive a.c. circuit.

(*iii*) In a pure capacitor, voltage lags behind the current by a phase angle of $\pi/2$.

$$V = V_0 \sin \omega t, \text{ and } I = I_0 \sin(\omega t + \pi/2) = I_0 \cos \omega t$$

Instantaneous power,

$$P = VI = V_0 I_0 \sin \omega t \cos \omega t = \frac{V_0 I_0}{2} \sin 2\omega t$$

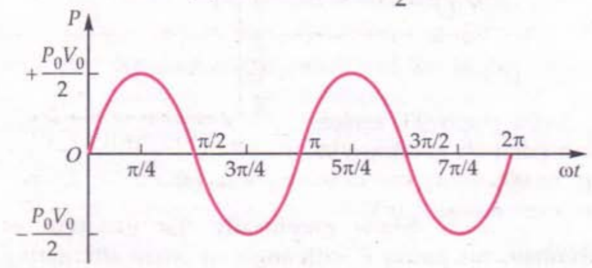


Fig. 7.65 (*c*) Variation of instantaneous power with ωt in a capacitive a.c. circuit.

Problem 20. **On the basis of power dissipation in a.c. circuit, distinguish between resistance, reactance and impedance.**

Solution. (*i*) Power dissipation is maximum across a resistance.

(*ii*) Power dissipation is zero across reactance X_L or X_C.

(*iii*) Power dissipation across impedance Z depends on power factor $\cos \phi$ in accordance with the relation

$$P_{av} = V_{rms} \cdot I_{rms} \cdot \cos \phi.$$

Problem 21. **At an airport, a person is made to walk through the doorway of a metal detector, for security reasons. If she/he is carrying anything made of metal, the metal detector emits a sound. On what principle does this detector work ?**

Solution. **Principle of a metal detector.** A metal detector works on the principle of resonance in a.c. circuits. When we walk through a metal detector, we are, in fact, walking through a coil of many turns. The coil is connected to a capacitor tuned so that the circuit is in resonance. When we walk through with some metal in our pocket, the impedance of the circuit changes – resulting in significant change in current in the circuit. This change in current is detected and the electronic circuit sounds as an alarm.

Problem 22. **A radio frequency choke is air-cored whereas an audio frequency choke is iron-cored. Give reason for this difference.** [CBSE D 97]

Solution. Inductive reactance of a coil of inductance L is given by $X_L = 2\pi f L$

Radio frequency is a high frequency. To maintain X_L low, L should have a small value. This is obtained by using an air-cored choke for which $\mu = 1$.

On the other hand, audio frequency is a low frequency. To maintain a sufficient value of X_L, L should have a large value. This is obtained by using iron core for which μ has a large value.

Problem 23. **(*a*) Obtain the expression for the magnetic energy stored in a solenoid in terms of magnetic field B, area A and length l of the solenoid. (*b*) How does this magnetic energy compare with the electrostatic energy stored in a capacitor ?** [NCERT ; CBSE D 15C]

Or

Starting from the expression for the energy $W = \frac{1}{2} LI^2$, stored in a solenoid of self-inductance L to build up the current I, obtain the expression for the magnetic energy in terms of the magnetic field B, area A and length l of the solenoid having n number of turns per unit length. Hence show that the energy density is given by $B^2/2\mu_0$. [CBSE D 13C]

Solution. The work done against the induced emf in building up the current from 0 to I is

$$W = \int dW = \int \varepsilon I \, dt = \int L \frac{dI}{dt} I dt = \int_0^I L I dI = \frac{1}{2} L I^2$$

The magnetic energy stored in the solenoid is

$$U_B = W = \frac{1}{2} LI^2 = \frac{1}{2} L \left(\frac{B}{\mu_0 n} \right)^2 \quad [\because B = \mu_0 n I]$$

$$= \frac{1}{2} (\mu_0 n^2 A l) \left(\frac{B}{\mu_0 n} \right)^2 \quad [\because L = \mu_0 n^2 A l]$$

or $$U_B = \frac{1}{2\mu_0} B^2 A l$$

(*b*) The magnetic energy stored per unit volume of the solenoid is

$$u_B = \frac{U_B}{V} = \frac{U_B}{Al}$$

[Here V is volume that contains flux]

or $$u_B = \frac{B^2}{2\mu_0} \quad ...(1)$$

We know that the electrostatic energy stored per unit volume in a parallel plate capacitor is

$$u_E = \frac{1}{2} \varepsilon_0 E^2 \quad ...(2)$$

In both the cases energy is proportional to the square of the field strengths. Eqs. (1) and (2) are general and valid for any region of space in which a magnetic field or/and an electric field exist.

Problem 24. **Give two disadvantages of transmitting a.c. over long distances at low voltage and high current.** [CBSE OD 96C]

Solution. Following are the disadvantages of transmitting electrical power at low voltage :

1. Large lengths of transmission cables have sufficient resistance. Hence a large amount of energy (I^2Rt) will be lost as heat during transmission.
2. Large voltage drop (IR) occurs along the line wire. Hence the voltage at the receiving station will be much smaller than that at the generating station.

Problem 25. **11 kilowatts of power can be transmitted in two ways :**

(*i*) 220 volts at 50 amperes and

(*ii*) 22,000 volts at 0.5 ampere.

Which is economical ? Give reasons for your choice. [CBSE F 11]

Solution. Let R be the resistance of the transmission line

(*i*) When 11 kW power is transmitted at 220 V at 50 A, the line loss is

$$I^2 R = (50)^2 \times R = 2500\ R$$

(*ii*) When 11 kW power is transmitted at 22,000 V at 0.5 A, the line loss is

$$I^2 R = (0.5)^2 R = 0.25\ R$$

Thus the line loss in case (*ii*) is much less than that in case (*i*). It is more economical to supply 11 kW power at 22,000 V at 0.5 A.

Problem 26. **(*a*) Out of the arrangements, given below for winding of primary and secondary coils in a transformer, which arrangement do you think will have higher efficiency and why ?**

(*b*) Show that in an ideal transformer, when the voltage is stepped up by a certain factor, the current gets stepped down by the same factor.

(*c*) State any two causes of energy loss in a transformer. [CBSE Sample Paper 11]

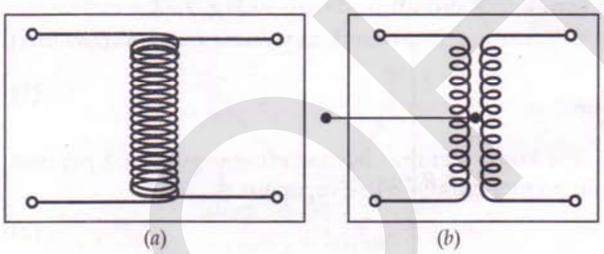


Fig. 7.66

Solution. (*a*) Arrangement (*a*) will have a higher efficiency, because leakage of flux is minimum (almost zero) when the primary envelopes the secondary.

(*b*) For an ideal transformer,

Input power = Output power

$$\mathcal{E}_p I_p = \mathcal{E}_s I_s$$

$$\therefore \quad \frac{\mathcal{E}_s}{\mathcal{E}_p} = \frac{I_p}{I_s}$$

So, when $\mathcal{E}_s$ increases, I_s decreases in the same ratio.

(*c*) The reasons for energy loss in a transformer are :

1. Resistance of the windings due to which energy is lost as heat ($I^2 R$).
2. Flux leakage because all the flux of primary does not pass through the secondary.

Problem 27. **Box' *A*, in the set up shown in Fig. 7.67, represents an electric device often used/needed to supply, electricity power from the (ac) mains, to a load. It is known that $V_o < V_i$.**

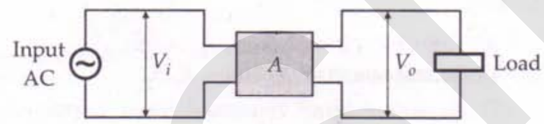


Fig. 7.67

(*a*) Identify the device A and draw its symbol.

(*b*) Draw a schematic diagram of this electric device. Explain its principle and working. Obtain an expression for the ratio between its output and input voltages.

(*c*) Find the relation between the input and output currents of this device assuming it to be ideal. [CBSE Sample Paper 13]

Solution. (*a*) The device is a step-down transformer. Its symbol is shown in Fig. 7.68.

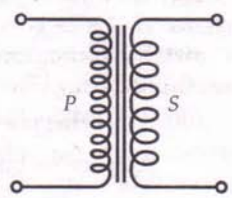


Fig. 7.68

(*b*) For transformer, refer answer to Q. 32 on page 7.43.

(*c*) Refer to the solution of part (*b*) of the above problem.

Problem 28. **A simple a.c. generator having a constant magnetic field is connected to a resistive load. Explain with reasons what will be the effects of doubling its speed of rotation on the following :**

(*a*) the frequency of rotation,

(*b*) the generated emf, and

(*c*) the mechanical power required to rotate the generator ? [CBSE F 95]

Solution. The maximum emf induced in a generator is given by

$$\mathcal{E}_0 = NBA\omega$$

When speed of rotation (ω) is doubled,

(*a*) Frequency of a.c. will be doubled.

(*b*) emf gets doubled.

(*c*) Mechanical power required to rotate the generator also gets doubled.

HOTS Problems on Higher Order Thinking Skills

Problem 1. *Two bulbs B_1 and B_2 are connected in series with an a.c. source of emf 200 V, as shown in Fig. 7.69. The labels on the bulbs read 200 V, 60 W and 200 V, 100 W respectively.*

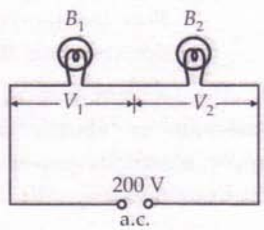


Fig. 7.69

Calculate the ratio of :

(*i*) *the resistances of the bulbs, R_1/R_2.*

(*ii*) *the power being consumed when connected in series, P_1/P_2.*

(*iii*) *the p.d. across the bulbs, V_1/V_2.* **[ISCE 03]**

Solution. (*i*) Here $P_1 = 60$ W, $P_2 = 100$ W

$\therefore$ Now $R_1 = \dfrac{V^2}{P_1}$ and $R_2 = \dfrac{V^2}{P_2}$

$$\therefore \quad \frac{R_1}{R_2} = \frac{P_2}{P_1} = \frac{100}{60} = \mathbf{5:3}.$$

(*ii*) In a series circuit, current I is same

$$\therefore \quad P_1 = I^2 R_1 \text{ and } P_2 = I^2 R_2$$

Hence $\dfrac{P_1}{P_2} = \dfrac{R_1}{R_2} = \mathbf{5:3}.$

(*iii*) P.D. across bulb B_1, $V_1 = IR_1$

P.D. across bulb B_2, $V_2 = IR_2$

$$\therefore \quad \frac{V_1}{V_2} = \frac{IR_1}{IR_2} = \frac{R_1}{R_2} = \mathbf{5:3}.$$

Problem 2. *You are given three circuit elements X, Y and Z. When the element X is connected across an a.c. source of a given voltage, the current and the voltage are in the same phase. When the element Y is connected in series with X across the source, voltage is ahead of the current in phase by $\pi/4$. But the current is ahead of the voltage in phase by $\pi/4$ when Z is connected in series with X across the source. Identify the circuit elements X, Y and Z.*

When all the three elements are connected in series across the same source, determine the impedance of the circuit.

Draw a plot of the current versus the frequency of applied source and mention the significance of this plot. **[CBSE OD 15]**

Solution. X is a resistor, Y is an inductor, C is a capacitor.

Clearly, $X_L = X_C$, so the impedance of the series circuit is $Z = \sqrt{R^2 + (X_L - X_C)^2} = R$

Fig. 7.70 shows the plot of current versus frequency of a.c. source. At the resonant frequency, $f = f_r$, the current is maximum. Sharpness of the current peak indicates the sharpness of resonance or Q-factor.

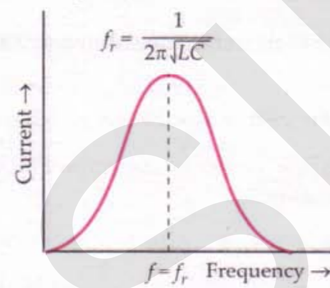


Fig. 7.70

Problem 3. *Mention the factors on which the resonant frequency of a series LCR-circuit depends. Plot a graph showing variation of impedance of a series LCR-circuit with the frequency of the applied a.c. source.* **[CBSE OD 05]**

Solution. The resonant frequency of a series LCR-circuit depends on the values of inductance L and capacitance C. In fact,

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

The variation of impedance Z with frequency f for a series LCR-circuit is shown in Fig. 7.71.

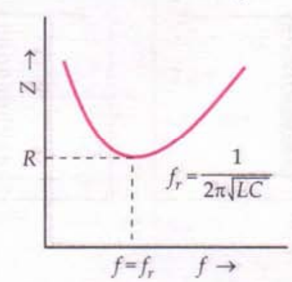


Fig. 7.71

Problem 4. *Fig. 7.72 shows a light bulb (B) and an iron cored inductor connected to a DC battery through a switch (S).*

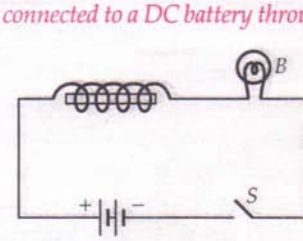


Fig. 7.72

(*i*) *What will one observe when switch (S) is closed ?*

(ii) How will the glow of the bulb change when the battery is replaced by an a.c. source of rms voltage equal to the voltage of DC battery ? Justify your answer in each case. **[CBSE SP 08]**

Solution. (*i*) When the switch S is closed, the glow of the bulb gradually increases till it becomes maximum.

Reason. As the current starts growing, a back emf $\left(\varepsilon = -L\frac{dI}{dt}\right)$ is induced in the inductor. This delays the growth of current to its final steady value in the circuit.

(*ii*) The glow will decrease.

Reason. When the ac source replaces the battery, the iron-cored inductor begins to offer a large reactance. The impedance of the circuit increases and current decreases.

Problem 5. *When a circuit element 'X' is connected across an a.c. source, a current of $\sqrt{2}$ A flows through it and this current is in phase with the applied voltage. When another element 'Y' is connected across the same a.c. source, the same current flows in the circuit but it leads the voltage by $\pi/2$ radians.*

(*i*) *Name the circuit elements X and Y.*

(*ii*) *Find the current that flows in the circuit when the series combination of X and Y is connected across the same a.c. voltage.*

(*iii*) *Plot a graph showing variation of the net impedance of this series combination of X and Y as a function of the angular frequency of the applied voltage.*

[CBSE Sample Paper 08]

Solution. (*i*) The circuit element X is a resistor and Y is a capacitor.

(*ii*) Here $R = X_C = \frac{V_{eff}}{\sqrt{2}}\ \Omega$

When X and Y are connected in series, the impedance becomes

$$Z = \sqrt{R^2 + X_C^2} = \sqrt{2\ R^2} = \sqrt{2}\ R$$

Current, $I_{eff} = \frac{V_{eff}}{Z} = \frac{V_{eff}}{\sqrt{2}\ R} = \frac{\sqrt{2}\ R}{\sqrt{2}\,R} = \mathbf{1\ A}.$

(*iii*) Impedance, $Z = \sqrt{R^2 + X_C^2} = \sqrt{R^2 + \frac{1}{\omega^2 C^2}}$

The Z-ω graph will be of the type as shown in Fig. 7.73.

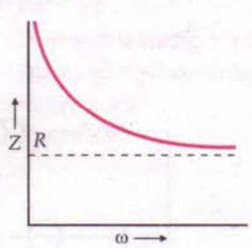


Fig. 7.73

Problem 6. *Three students X, Y, Z performed an experiment for studying the variation of alternating currents with angular frequency in a series LCR-circuit and obtained the graphs shown below. They all used a.c. sources of the same rms value and inductances of the same value.*

What can we (qualitatively) conclude about the

(i) capacitance values

(ii) resistance

used by them ? In which case will the quality factor be maximum ?

What can we conclude about nature of the impedance of the setup at the frequency ω_0 ? **[CBSE Sample Paper 08]**

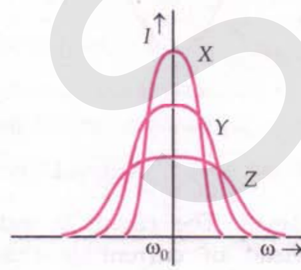


Fig. 7.74

Solution. (*i*) Clearly, the resonant frequency ω_0 is same for all the three graphs X, Y and Z.

As $\omega_0 = \frac{1}{\sqrt{LC}}$ and $L_X = L_Y = L_Z$, so $C_X = C_Y = C_Z$.

(*ii*) The maximum value of current at resonance is

$$I_0 = \frac{\varepsilon_0}{R} \quad i.e., \quad I_0 \propto \frac{1}{R}$$

But $\quad I_0^X > I_0^Y > I_0^Z$

$\therefore \quad R_X < R_Y < R_Z$

Quality factor, $Q = \frac{\omega L}{R}$

As ω and L are same in all three cases, so $Q \propto \frac{1}{R}$

Now $R_X < R_Y < R_Z$, so $Q_X > Q_Y > Q_Z$

i.e., Q is maximum in case X.

At resonance frequency ω_0, $X_L = X_C$, so

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = R,$$

Thus the impedance of the circuit is purely resistive in nature.

Problem 7. *For a series LCR circuit, connected to a sinusoidal a.c. voltage source, identify the graph that corresponds to $\omega > \frac{1}{\sqrt{LC}}$. Give reason.* **[CBSE OD 07C]**

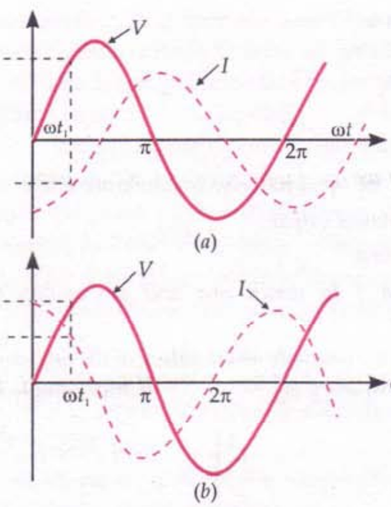


Fig. 7.75

Solution. For $\omega > \frac{1}{\sqrt{LC}}$, $X_L (= \omega L)$ is large and $X_C (= 1/\omega C)$ is small. The circuit is inductive. So voltage V is ahead of current I. This situation corresponds to graph (*a*).

Problem 8. *The graphs shown below, depict the variation of current i_m vs. angular frequency (ω) for two different series LCR-circuits.*

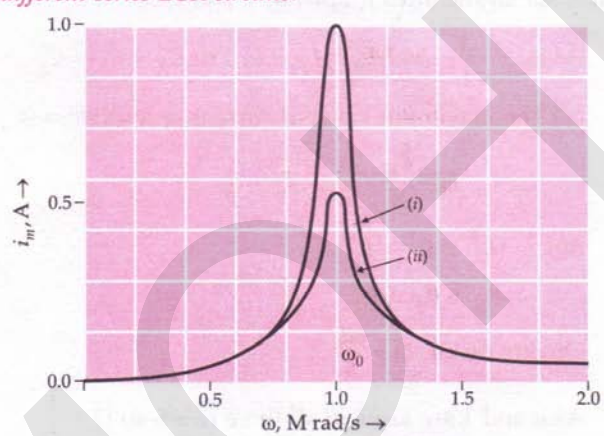


Fig. 7.76

Observe the graphs carefully :

(*i*) *State the relation between the L and C values of the two circuits, when the current in the two circuits is maximum.*

(*ii*) *Indicate the circuit for which*

(*a*) *power factor is higher*

(*b*) *quality factor (Q) is larger.*

Give reasons for each case. **[CBSE D 09C]**

Solution. (*i*) Clearly, the current in the two *LCR*-circuits is maximum for the same angular frequency, $\omega_0 = 1.0$ Mrad/s.

For each circuit,

$$\omega_0 = \frac{1}{\sqrt{LC}} \quad \text{or} \quad L_1 C_1 = L_2 C_2$$

(*ii*) (*a*) At resonant frequency ω_0,

$$Z = R$$

So, power factor for both circuits is same and is equal to unity.

$$\cos\phi = \frac{R}{Z} = 1$$

(*b*) As the resonance peak is sharper for circuit (*i*), so it has a larger *Q*-factor.

Problem 9. *Figure 7.77 shows how the reactance of an inductor varies with frequency.*

(*i*) *Calculate the value of the inductance of the inductor using the information given in the graph.*

(*ii*) *If this inductor is connected in series to a resistor of 8 ohm, find what would be the impedance at 300 Hz ?* **[CBSE OD 03C]**

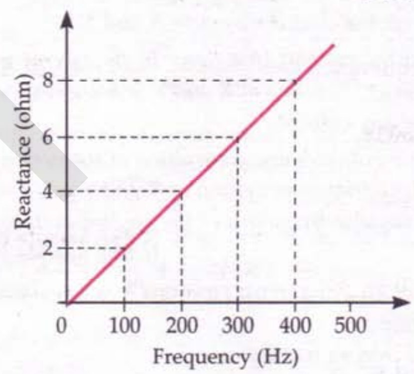


Fig. 7.77

Solution. (*i*) Inductance,

$$L = \frac{X_L}{2\pi f}$$

$$= \frac{1}{2\pi} \times \text{slope of } X_L - f \text{ graph}$$

$$= \frac{1}{2\pi} \times \frac{8-0}{400-0} = \frac{1}{100\pi} = \mathbf{3.18 \times 10^{-3}\,H.}$$

(*ii*) From the given graph, when $f = 300$ Hz, $X_L = 6\,\Omega$

$\therefore$ Impedance, $Z = \sqrt{R^2 + X_L^2} = \sqrt{8^2 + 6^2} = \mathbf{10\,\Omega}$.

Problem 10. *In the circuit shown in Fig. 7.78, the current is found to lag behind the voltage by an angle of 36.9°.*

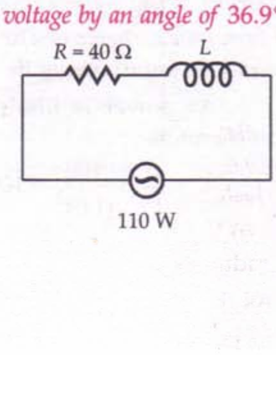


Fig. 7.78

Calculate the

(*i*) *inductive reactance,*
(*ii*) *impedance of the circuit,*
(*iii*) *current flowing in the circuit, and*
(*iv*) *frequency of the applied emf.*

Take $L = 0.1$ *H,* $\cos 36.9° = 4/5$ *and* $\tan 36.9° = 3/4$. **[ISCE 02]**

Solution. (*i*) As $\tan\phi = \frac{X_L}{R}$

$$\therefore \quad X_L = R\tan\phi = 40\tan 36.9°$$
$$= 40 \times \frac{3}{4} = \mathbf{30\ \Omega}.$$

(*ii*) Impedance,

$$Z = \sqrt{R^2 + X_L^2}$$
$$= \sqrt{40^2 + 30^2} = \mathbf{50\ \Omega}.$$

(*iii*) $I_{rms} = \frac{\mathcal{E}_{rms}}{Z} = \frac{110}{50} = \mathbf{2.2\ A}$.

(*iv*) Frequency, $f = \frac{X_L}{2\pi L} = \frac{30}{2\pi \times 0.1} = \mathbf{47.75\ A}$.

Problem 11. *A 50 W – 100 V electric bulb is to be used on a 200 V – 50 Hz a.c. supply. Calculate the inductance of the lamp so that it may glow with its normal brightness. (Take* $\pi = 3$*)* **[CBSE Sample Paper 03]**

Solution. $I_{rms} = \frac{P}{V} = \frac{50\text{ W}}{100\text{ V}} = \frac{1}{2}$ A

Voltage across inductance L,

$$V_L = \sqrt{V_{eff}^2 - V_R^2}$$
$$= \sqrt{(200)^2 - (100)^2} = 100\sqrt{3}\text{ V}$$

The bulb will glow with its normal brightness if the same current flows through it. Therefore, the reactance of inductance L,

$$X_L = \frac{V_L}{I_{rms}} = \frac{100\sqrt{3}}{1/2} = 200\sqrt{3}\ \Omega$$

Inductance,

$$L = \frac{X_L}{2\pi f} = \frac{200\sqrt{3}}{2 \times 3 \times 50} = \mathbf{1.155\ H}.$$

Problem 12. *A coil draws a current of 1.0 A and a power of 100 W from an a.c. source of 110 V and 50 Hz. Find the resistance and the inductance of the coil.*

Solution. As power is dissipated only by ohmic resistance, therefore

$$R = \frac{P}{I^2} = \frac{100}{(1.0)^2} = \mathbf{100\ \Omega}$$

As $$Z = \frac{\mathcal{E}_{eff}}{I_{eff}} = \sqrt{R^2 + X_L^2}$$

$$\therefore \quad \frac{110}{1.0} = \sqrt{100^2 + X_L^2}$$

or $$X_L = \sqrt{110^2 - 100^2} = \sqrt{2100} = 45.8\ \Omega$$

$$L = \frac{X_L}{2\pi f} = \frac{45.8}{2 \times 3.14 \times 50} = \mathbf{0.146\ H.}$$

Problem 13. *Figure 7.79 given below shows how the reactance of a capacitor varies with frequency.*

(*i*) *Use the information on graph to calculate the value of capacity of the capacitor.*
(*ii*) *An inductor of inductance 'L' has the same reactance as the capacitor at 100 Hz. Find the value of L.*
(*iii*) *Using the same axes, draw a graph of reactance against frequency for the inductor given in part (ii).*
(*iv*) *If this capacitor and inductor were connected in series to a resistor of 10 Ω, what would be the impedance of the combination at 300 Hz ?* **[CBSE D 03C]**

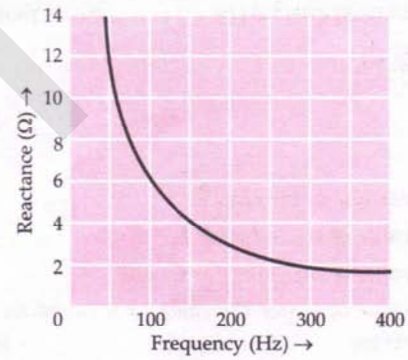


Fig. 7.79

Solution. (*i*) For $f = 100$ Hz, $X_C = 6\ \Omega$

As $$X_C = \frac{1}{2\pi f C}$$

$$\therefore \quad C = \frac{1}{2\pi f X_C} = \frac{1}{2\pi \times 100 \times 6}$$
$$= \mathbf{2.65 \times 10^{-4}\ F.}$$

(*ii*) For $f = 100$ Hz, $X_L = X_C = 6\ \Omega$

As $$X_L = 2\pi f L$$

$$\therefore \quad L = \frac{X_L}{2\pi f} = \frac{6}{2\pi \times 100} = \mathbf{9.459 \times 10^{-3}\ H.}$$

(*iii*) As $X_L \propto f$, so values of X_L at different values of f are as follows :

f(Hz)	100	200	300	400
X_L(Ω)	6	12	18	24

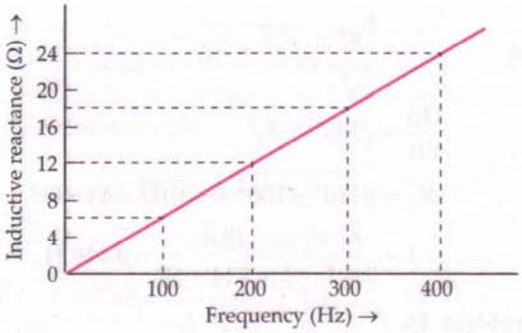


Fig. 7.80

(*iv*) Now

$$f' = 300 \text{ Hz}$$

$$X_C' = \frac{f}{f'} \cdot X_C = \frac{100}{300} \times 6 = 2\Omega$$

$$X_L' = \frac{f'}{f} \times X_L = \frac{300}{100} \times 6 = 18\,\Omega$$

$$Z = \sqrt{R^2 + (X_L' - X_C')^2}$$

$$= \sqrt{10^2 + (18-2)^2} = \sqrt{356} = \mathbf{18.87\,\Omega}.$$

Problem 14. *A series LCR-circuit is connected to an a.c. source (220 V – 50 Hz), as shown in Fig. 7.81. If the voltages of the three voltmeters V_1, V_2 and V_3 are 65 V, 415 V and 204 V respectively, calculate :*

(*i*) *the current in the circuit,*
(*ii*) *the value of the inductor L,*
(*iii*) *the value of the capacitor C, and*
(*iv*) *the value of C (for the same L) required to produce resonance.* **[ISCE 94]**

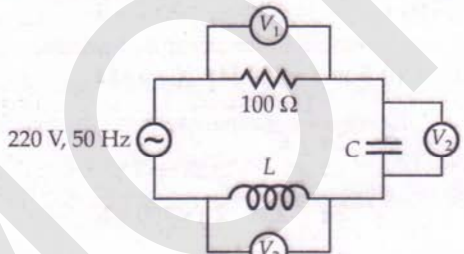


Fig. 7.81

Solution. Here $\mathcal{E}_{rms} = 220$ V, $f = 50$ Hz, $R = 100\,\Omega$, $V_R = 65$ V, $V_C = 415$ V, $V_L = 204$ V

(*i*) If I_{rms} be the current in the circuit, then

$$V_R = I_{rms}\, R$$

or $$I_{rms} = \frac{V_R}{R} = \frac{65}{100} = \mathbf{0.65\ A.}$$

(*ii*) As $V_L = I_{rms}\, X_L$

$$\therefore \quad X_L = \frac{V_L}{I_{rms}} = \frac{204}{0.65} = 313.85\,\Omega$$

or $$2\pi f L = 313.85\,\Omega$$

or $$L = \frac{313.85}{2\pi f} = \frac{313.85}{2\pi \times 50}$$

$$= \mathbf{1.0\ H.}$$

(*iii*) As $V_C = I_{rms}\, X_C$

$$\therefore \quad X_C = \frac{V_C}{I_{rms}} = \frac{415}{0.65} = \mathbf{638.46\,\Omega}.$$

But $$X_C = \frac{1}{2\pi f C}$$

$$\therefore \quad C = \frac{1}{2\pi f X_C} = \frac{1}{2\pi \times 50 \times 638.46}$$

$$= 5 \times 10^{-6}\ \text{F} = \mathbf{5\ \mu F}.$$

(*iv*) Suppose a capacitor of capacitance C′ produces resonance with an inductor of 1.0 H. Then

$$2\pi f L = \frac{1}{2\pi f C'}$$

or $$C' = \frac{1}{4\pi^2 f^2 L} = \frac{1}{4 \times 9.87 \times (50)^2 \times 1.0}$$

$$= 10.1 \times 10^{-6} = \mathbf{10.1\ \mu F}.$$

Problem 15. *Show that if a coil of self-inductance L and resistance R is connected to a source of emf, $\mathcal{E} = \mathcal{E}_0 \sin \omega t$, the average power consumed is $\frac{1}{2}\mathcal{E}_0^2 R/(R^2 + \omega^2 L^2)$.*

Solution. Given $\mathcal{E} = \mathcal{E}_0 \sin \omega t$

$$\therefore \quad I = I_0 \sin(\omega t - \phi), \quad \text{where} \quad \tan\phi = \frac{\omega L}{R}$$

The power is consumed only across the resistance and not across the inductance. So average power consumed per cycle is

$$P_{av} = \frac{1}{T}\int_0^T I^2 R\, dt = \frac{1}{T}\int_0^T I_0^2 \sin^2(\omega t - \phi) R\, dt$$

$$= \frac{I_0^2 R}{2T}\int_0^T 2\sin^2(\omega t - \phi)\, dt$$

$$= \frac{I_0^2 R}{2T}\int [1 - \cos 2(\omega t - \phi)]\, dt$$

$$= \frac{I_0^2 R}{2T}[T - 0] = \frac{\mathcal{E}_0^2 R}{2(R^2 + \omega^2 L^2)}$$

$$\left[\because I_0 = \frac{\mathcal{E}_0}{\sqrt{R^2 + \omega^2 L^2}}\right]$$

GUIDELINES TO NCERT EXERCISES

7.1. *A 100 Ω resistor is connected to a 220 V, 50 Hz ac supply.*

(a) What is the rms value of current in the circuit ?

(b) What is the net power consumed over a full cycle ?

Ans. Here, $R = 100\,\Omega$, $\mathcal{E}_{rms} = 220$ V, $f = 50$ Hz

(a) $I_{rms} = \frac{\mathcal{E}_{rms}}{R} = \frac{220}{100} = \mathbf{2.20\ A}$

(b) $P_{av} = \mathcal{E}_{rms} I_{rms} = 220 \times 2.2 = \mathbf{484\ W.}$

7.2. *(a) The peak value of an a.c. supply is 300 V. What is the rms voltage ?*

(b) The rms value of current in an a.c. circuit is 10 A. What is the peak current ?

Ans. (a) Here $\mathcal{E}_0 = 300$ V

$\therefore \quad \mathcal{E}_{rms} = 0.707\,\mathcal{E}_0 = 0.707 \times 300 = \mathbf{212.1\ V.}$

(b) Here $I_{rms} = 10$ A

$\therefore \quad I_0 = \sqrt{2}\, I_{rms} = 1.414 \times 10 = \mathbf{14.14\ A.}$

7.3. *A 44 mH inductor is connected to 220 V, 50 Hz a.c. supply. Determine the rms value of current in the circuit.*

Ans. Here $L = 44\text{ mH} = 44 \times 10^{-3}$ H, $\mathcal{E}_{rms} = 220$ V, $f = 50$ Hz

Reactance, $X_L = 2\pi f L = 2\pi \times 50 \times 44 \times 10^{-3}\,\Omega$

Current, $I_{rms} = \frac{\mathcal{E}_{rms}}{X_L} = \frac{220}{2\pi \times 50 \times 44 \times 10^{-3}} = \mathbf{15.9\ A.}$

7.4. *A 60 μF capacitor is connected to a 110 V, 60 Hz a.c. supply. Determine the rms value of current in the circuit.*

Ans. Here $C = 60\,\mu\text{F} = 60 \times 10^{-6}$ F,

$\mathcal{E}_{rms} = 110$ V, $f = 60$ Hz

Capacitive reactance,

$$X_C = \frac{1}{2\pi fC}$$

$$= \frac{1}{2 \times 3.14 \times 60 \times 60 \times 10^{-6}} = 44.2\,\Omega$$

$$I_{rms} = \frac{\mathcal{E}_{rms}}{X_C} = \frac{110}{44.2} = \mathbf{2.49\ A.}$$

7.5. *In Exercises 7.3 and 7.4, what is the net power absorbed by each circuit over a complete cycle ? Explain your answer.*

Ans. The current through an inductor lags behind the emf by $\pi/2$ rad while the current through a capacitor is ahead of the emf by $\pi/2$ rad. So in each case,

$$P_{av} = \mathcal{E}_{rms} I_{rms} \cos\left(\pm \frac{\pi}{2}\right) = \mathbf{0.}$$

7.6. *Obtain the resonant frequency ω_r of a series LCR-circuit with $L = 2.0$ H, $C = 32\,\mu$F and $R = 10\,\Omega$. What is the Q-value of the circuit ?*

Ans. Here $L = 2.0$ H, $C = 32\,\mu\text{F} = 32 \times 10^{-6}$ F, $R = 10\,\Omega$

Resonant frequency,

$$\omega_r = \frac{1}{\sqrt{LC}}$$

$$= \frac{1}{\sqrt{2.0 \times 32 \times 10^{-6}}} = \frac{1000}{8} = \mathbf{125\ rad\ s^{-1}}$$

$$Q\text{-value} = \frac{\omega_r L}{R} = \frac{125 \times 2.0}{10} = \mathbf{25.}$$

7.7. *A charged 30 μF capacitor is connected to a 27 mH inductor. What is the angular frequency of free oscillations of the circuit ?*

Ans. Here $C = 30 \times 10^{-6}$ F, $L = 27 \times 10^{-3}$ H

The angular frequency of free oscillations of the LC-circuit is

$$\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{27 \times 10^{-3} \times 30 \times 10^{-6}}}$$

$$= \frac{1}{9} \times 10^4 \text{ rad s}^{-1} = \mathbf{1.1 \times 10^3\ rad\ s^{-1}.}$$

7.8. *Suppose the initial charge on the capacitor in Exercise 7.7 is 6 mC. What is the total energy stored in the circuit initially ? What is the total energy at later time ?*

Ans. Here $C = 30 \times 10^{-6}$ F, $q_0 = 6 \times 10^{-3}$ C

Total energy stored in the inductor initially,

$$U = U_E^{\max} = \frac{1}{2}\frac{q_0^2}{C} = \frac{1}{2} \cdot \frac{(6 \times 10^{-3})^2}{30 \times 10^{-6}} = \frac{36}{60} = \mathbf{0.6\ J}$$

At later times, the energy is shared between C and L. However, the total energy remains constant (= 0.6 J), assuming that there is no loss of energy.

7.9. *A series LCR-circuit with $R = 20\,\Omega$, $L = 1.5$ H and $C = 35\,\mu$F is connected to a variable-frequency 200 V a.c. supply. When the frequency of the supply equals the natural frequency of the circuit, what is the average power transferred to the circuit in one complete cycle ?* [Haryana 01]

Ans. When the frequency of the a.c. source equals the natural frequency of the circuit, the impedance is

$$Z = R = 20\,\Omega$$

The average power dissipated per cycle,

$$P_{av} = \frac{\mathcal{E}_{rms}^2}{Z} = \frac{\mathcal{E}_{rms}^2}{R} = \frac{(200)^2}{20} = \mathbf{2000\ W.}$$

7.10. *A radio can tune over the frequency range of a portion of MW (medium wave) broadcast band : (800 kHz to 1200 kHz). If its LC circuit has an effective inductance of 200 μ H, what must be the range of its variable condenser ?*

Ans. For tuning, the frequency of free *LC*-oscillations should be equal to the frequency of the radiowave. The value of this frequency is

$$f = \frac{1}{2\pi\sqrt{LC}} \quad \text{or} \quad C = \frac{1}{4\pi^2 f^2 L}$$

(*i*) For $f = 800 \text{ kHz} = 800 \times 10^3$ Hz

$$C = \frac{1}{4\pi^2 \times (800 \times 10^3)^2 \times 200 \times 10^{-6}}$$

$$= 197.8 \times 10^{-12}\,\text{F} \simeq \mathbf{198\ pF}$$

$[\because 1\,\text{pF} = 10^{-12}\,\text{F}]$

(*ii*) For $f = 1200 \text{ kHz} = 1200 \times 10^3$ Hz,

$$C = \frac{1}{4\pi^2 \times (1200 \times 10^3)^2 \times 200 \times 10^{-6}}\,\text{F}$$

$$= 87.9 \times 10^{-12}\,\text{F} \simeq \mathbf{88\ pF}$$

Thus the variable capacitor should have a range of about 88 pF to 198 pF.

7.11. *Figure 7.82 shows a series LCR-circuit connected to a variable frequency 230 V source. L = 5.0 H, C = 80 μF, R = 40 Ω.*

(*a*) *Determine the source frequency which drives the circuit in resonance.*

(*b*) *Obtain the impedance of the circuit and the amplitude of current at the resonating frequency.*

(*c*) *Determine the rms potential drops across the three elements of the circuit. Show that the potential drop across the LC- combination is zero at the resonating frequency.* [CBSE D 06]

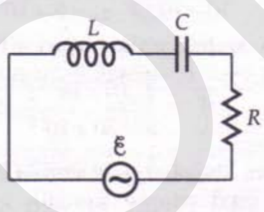


Fig. 7.82

Ans. Here $V_{rms} = 230$ V, $L = 5.0$ H

$C = 80\,\mu\text{F} = 80 \times 10^{-6}\,\text{F}$, $R = 40\,\Omega$

(*a*) The resonant angular frequency is

$$\omega_r = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(5 \times 80 \times 10^{-6})}} = \mathbf{50\ rad\ s^{-1}}.$$

(*b*) At $\omega = \omega_r$, $\omega L = \dfrac{1}{\omega C}$, therefore,

Impedance, $$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} = R = \mathbf{40\,\Omega}$$

Current amplitude

$$= \frac{V_0}{Z} = \frac{V_0}{R} \text{ (at resonance)}$$

$$= \frac{\sqrt{2}\,V_{rms}}{R} = \frac{1.414 \times 230}{40} = \mathbf{8.1\ A.}$$

(*c*) $$I_{rms} = \frac{V_{rms}}{R} = \frac{230}{40} = \frac{23}{4}\,\text{A}$$

∴ P.D. across *L*,

$$V_{rms}^L = I_{rms} \times \omega_r L = \frac{23}{4} \times 50 \times 5 = \mathbf{1437.5\ V}$$

P.D. across *C*,

$$V_{rms}^C = I_{rms} \times \frac{1}{\omega_r C}$$

$$= \frac{23}{4} \times \frac{1}{50 \times 80 \times 10^{-6}} = \mathbf{1437.5\ V}$$

P.D. across the *LC*-combination at the resonant frequency

$$= I_{rms}\left(\omega_r L - \frac{1}{\omega_r C}\right) = 0$$

P.D. across *R*,

$$V_{rms}^R = I_{rms} \times R = \frac{23}{4} \times 40 = \mathbf{230\ V}$$

= Applied rms voltage, as expected.

This is because the total potential drop across the *LC*-combination is zero.

7.12. *An LC-circuit contains a 20 mH inductor and a 50 μF capacitor with an initial charge of 10 mC. The resistance of the circuit is negligible. Let the instant the circuit is closed be t = 0.*

(*a*) *What is the total energy stored initially. Is it conserved during the LC-oscillations ?*

(*b*) *What is the natural frequency of the circuit ?*

(*c*) *At what times is the energy stored ?*

(i) completely electrical (i.e., stored in the capacitor) ?

(ii) completely magnetic (i.e., stored in the inductor) ?

(*d*) *At what times is the total energy shared equally between the inductor and the capacitor ?*

(*e*) *If a resistor is inserted in the circuit, how much energy is eventually dissipated as heat ?* [CBSE Sample Paper 98]

Ans. Here $L = 20\,\text{mH} = 20 \times 10^{-3}\,\text{H}$;

$C = 50\,\mu\text{F} = 50 \times 10^{-6}\,\text{F}$

Initial charge on capacitor,

$$q_0 = 10\,\text{mC} = 10 \times 10^{-3}\,\text{C}$$

(*a*) Total energy stored initially

$$= \frac{q_0^2}{2C} = \frac{(10^{-2})^2}{2 \times 50 \times 10^{-6}}\,\text{J} = \mathbf{1\ J}$$

Yes, the total energy is conserved in *LC*-oscillations because the resistance of the *LC*-circuit is negligible.

(*b*) The natural frequency of the circuit is

$$f = \frac{1}{2\pi\sqrt{LC}}$$

$$= \frac{1}{2 \times 3.14 \times \sqrt{20 \times 10^{-3} \times 50 \times 10^{-6}}} \text{ Hz}$$

or $$f = \frac{1}{2 \times 3.14 \times 10^{-3}} = 159.2 \simeq \mathbf{159\ Hz.}$$

(*c*) The charge on a capacitor plate at any instant during *LC*-oscillations is

$$q = q_0 \cos \omega t = q_0 \cos \frac{2\pi t}{T}$$

(*i*) The energy stored will be completely electrical when

$$q = \pm q_0$$

or $$\cos \frac{2\pi t}{T} = \pm 1$$

or $$\frac{2\pi t}{T} = n\pi, \text{ where } n \text{ is an integer}$$

or $$t = \frac{n}{2} T \text{ or } t = 0, \frac{T}{2}, T, \frac{3T}{2}, \ldots$$

(*ii*) The energy stored is completely magnetic when the electrical energy is zero or when

$$q = q_0 \cos \omega t = q_0 \cos \frac{2\pi t}{T} = 0$$

or $$\frac{2\pi t}{T} = (2n+1)\frac{\pi}{2}$$

or $$t = (2n+1)\frac{T}{4} \text{ or } t = \frac{T}{4}, \frac{3T}{4}, \frac{5T}{4}, \ldots$$

In both cases,

$$T = \frac{1}{f} = \frac{1}{159} \text{ s} = 6.28 \times 10^{-3} \text{ s} \simeq \mathbf{6.3\ ms.}$$

(*d*) Total energy $= \frac{1}{2}\frac{q_0^2}{C}$

Let q be the charge on the capacitor at the instants when the energy of capacitor becomes half of the total energy. At these instants, energy of the capacitor

$$= \frac{1}{2}\frac{q^2}{C}$$

$$\therefore \quad \frac{1}{2} \cdot \frac{q^2}{C} = \frac{1}{2}\left(\frac{1}{2}\frac{q_0^2}{C}\right) \text{ or } q = \pm \frac{q_0}{\sqrt{2}}$$

But $$q = q_0 \cos \omega t = q_0 \cos \frac{2\pi t}{T}$$

$$\therefore \quad \pm \frac{q_0}{\sqrt{2}} = q_0 \cos \frac{2\pi t}{t}$$

or $$\cos \frac{2\pi t}{T} = \pm \frac{1}{\sqrt{2}} = \pm \cos \frac{\pi}{4}$$

$$= \cos \frac{\pi}{4} \text{ or } \cos \frac{3\pi}{4}$$

or $$\frac{2\pi t}{T} = n\pi + \left(\frac{\pi}{4} \text{ or } \frac{3\pi}{4}\right)$$

or $$t = (4n+1)\frac{T}{8} \text{ or } (4n+3)\frac{T}{8}$$

or $$t = \frac{T}{8}, \frac{3T}{8}, \frac{5T}{8}, \frac{7T}{8}, \ldots$$

(*e*) R damps out the *LC*-oscillations eventually. The whole of the initial energy = 1 J is finally lost as heat.

7.13. *A coil of inductance 0.50 H and resistance 100 Ω is connected to a 240 V, 50 Hz a.c. supply.*

(a) What is the maximum current in the coil ?

(b) What is the time lag between the voltage maximum and the current maximum ?

Ans. For an *LR*-circuit, if $V = V_0 \cos \omega t$, then

$$I = \frac{V_0}{\sqrt{R^2 + \omega^2 L^2}} \cos(\omega t - \phi),$$

where $$\tan \phi = \frac{\omega L}{R}.$$

(*a*) Maximum current in the coil is

$$I_0 = \frac{V_0}{\sqrt{R^2 + \omega^2 L^2}} = \frac{V_0}{\sqrt{R^2 + 4\pi^2 f^2 L^2}}$$

Given $L = 0.50$ H, $R = 100\,\Omega$, $V_{eff} = 240$ V

and $f = 50$ Hz.

$$\therefore I_0 = \frac{\sqrt{2} \times 240}{\sqrt{(100)^2 + 4\pi^2 \times (50)^2 \times (0.50)^2}} \text{ A} \left[\because V_{eff} = \frac{V_0}{\sqrt{2}}\right]$$

$$= \frac{1.414 \times 240}{\sqrt{10000 + 24674}} = \frac{1.414 \times 240}{186.2} = \mathbf{1.82\ A}$$

(*b*) V is maximum at $t = 0$, I is maximum at $t = \frac{\phi}{\omega}$ (*i.e.*, when $\omega t - \phi = 0$). If ϕ is positive, this means current maximum lags behind voltage maximum by time lag,

$$\Delta t = \frac{\phi}{\omega}$$

Now $$\tan \phi = \frac{2\pi f L}{R} = \frac{2\pi \times 50 \times 0.5}{100} = 1.571$$

$$\therefore \quad \phi = \tan^{-1}(1.571) \simeq 57.5° = \frac{57.5\pi}{180} \text{ rad}$$

Time lag, $$\Delta t = \frac{\phi}{\omega} = \frac{57.5\pi}{180 \times 2\pi \times 50} \text{ s}$$

$$= 3.19 \times 10^{-3} \text{ s} \simeq \mathbf{3.2\ ms}$$

7.14. *Obtain the answers to (a) and (b) in Exercise 7.13, if the circuit is connected to a high frequency supply (240 V, 10 kHz). Hence explain statement that at very high frequency, inductor in circuit amounts to open circuit. How does an inductor behave in a d.c. circuit after the steady state ?*

Ans. Here $f = 10$ kHz $= 10^4$ Hz,

$$\omega = 2\pi f = 2\pi \times 10^4 \text{ rad s}^{-1}, \quad \mathcal{E}_{rms} = 240 \text{ V}$$

$$I_0 = \frac{\mathcal{E}_0}{\sqrt{R^2 + \omega^2 L^2}} = \frac{\sqrt{2} \times 240}{\sqrt{10^4 + 4\pi^2 \times 10^8 \times 0.5^2}}$$

$$= \mathbf{1.08 \times 10^{-2}\ A}$$

Here the contribution of the resistance R is negligible as compared to the reactance ωL.

Also, $\tan\phi = \frac{\omega L}{R} = \frac{2\pi \times 10^4 \times 0.5}{100} = 100\,\pi$

which is very large. So ϕ is nearly equal to $\pi/2$ rad.

Thus we see that I_0 is much smaller (1.08×10^{-2} A) than its value (1.82 A) at high frequency. At high frequency, L nearly amounts to an open circuit *i.e.*, it offers very large resistance. In a d.c. circuit (after attaining steady state) $\omega = 0$, so L acts like a pure conductor.

7.15. *A 100 μF capacitor in series with a 40 Ω resistance is connected to a 110 V, 60 Hz supply.*

(a) What is the maximum current in the circuit ?

(b) What is the time lag between current maximum and voltage maximum ?

Ans. For a CR-circuit, if $V = V_0 \cos \omega t$, then

$$I = \frac{V_0}{\sqrt{R^2 + \frac{1}{\omega^2 C^2}}} \cos(\omega t + \phi)$$

where $\tan\phi = \frac{1}{\omega CR}$

Here $V_{eff} = 110$ V, $\omega = 2\pi f = 2\pi \times 60$ rad s^{-1},

$R = 40\,\Omega$, $C = 100\,\mu F = 10^{-4}$ F

(*a*) Maximum current in the circuit is

$$I_0 = \frac{V_0}{\sqrt{R^2 + \frac{1}{\omega^2 C^2}}} = \frac{\sqrt{2}\,V_{eff}}{\sqrt{R^2 + \frac{1}{\omega^2 C^2}}}$$

$$= \frac{1.414 \times 110}{\sqrt{40^2 + \frac{1}{(2\pi \times 60 \times 10^{-4})^2}}}$$

$$= \frac{1.414 \times 110}{\sqrt{1600 + 703.62}} \simeq \frac{155.54}{48} = \mathbf{3.24\ A.}$$

(*b*) The phase angle ϕ is given by

$$\tan\phi = \frac{1}{\omega CR} = \frac{1}{2\pi \times 60 \times 10^{-4} \times 40} = 0.6631$$

$$\therefore \quad \phi = 33.5^\circ = \frac{33.5\,\pi}{180} \text{ rad}$$

Time lag, $\Delta t = \frac{\phi}{\omega} = \frac{33.5\,\pi}{180 \times 2\pi \times 60}$

$$= 1.55 \times 10^{-3}\text{s} = \mathbf{1.55\ ms.}$$

Here the voltage lags behind the current or the current leads the voltage.

7.16. *Obtain the answers to (a) and (b) in Exercise 7.15 if the circuit is connected to a 110 V, 12 kHz supply. Hence, explain the statement that a capacitor is a conductor at very high frequencies. Compare this behaviour with that of a capacitor in a d.c. circuit after the steady state.*

Ans. Here $R = 40\,\Omega$, $C = 100\,\mu F = 10^{-4}$ F, $\mathcal{E}_{rms} = 110$ V, $f = 12$ kHz $= 12 \times 10^3$ Hz

(*a*) $X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi \times 12 \times 10^3 \times 10^{-4}} = 0.133\,\Omega$

$$I_{rms} = \frac{\mathcal{E}_{rms}}{\sqrt{R^2 + X_C^2}} = \frac{110}{\sqrt{40^2 + (0.133)^2}} = 2.75 \text{ A}$$

$$\therefore \quad I_0 = \sqrt{2}\,I_{rms} = 1.414 \times 2.75 = \mathbf{3.89\ A.}$$

(*b*) $\tan\phi = \frac{X_C}{R} = \frac{0.133}{40} = 0.0033$

or $\phi = 0.2^\circ \simeq \mathbf{0^\circ}$

Now in the absence of capacitor,

$$I_{rms} = \frac{\mathcal{E}_{rms}}{R} = \frac{110}{40} = 2.75 \text{ A}$$

Hence at very high frequency (12 kHz), the current in the circuit is same both in the presence or absence of the capacitor. It follows that at high frequency, capacitor acts like a conductor.

For a d.c. supply, $f = 0$, so $X_C = \frac{1}{2\pi fC} = \infty$

Hence in a d.c. circuit, a capacitor amounts to an open circuit *i.e.*, it offers a very high resistance.

7.17. *Keeping the source frequency equal to the resonating frequency of the series LCR-circuit, if the three elements L, C and R are arranged in parallel, show that the total current in the parallel LCR-circuit is a minimum at this frequency. Obtain the current rms value in each branch of the circuit for L = 5.0 H, C = 80 μF and R = 40 Ω and for the a.c. source of emf 230 V for this frequency.*

Ans. The effective impedance of the parallel LCR combination is given by

$$\frac{1}{Z} = \frac{1}{R} + \frac{1}{j\,\omega L} + \frac{1}{\frac{1}{j\,\omega C}}$$

or $$\frac{1}{Z} = \frac{1}{R} + j\left(\omega C - \frac{1}{\omega L}\right)$$

or $$\frac{1}{|Z|} = \sqrt{\frac{1}{R^2} + \left(\omega C - \frac{1}{\omega L}\right)^2}$$

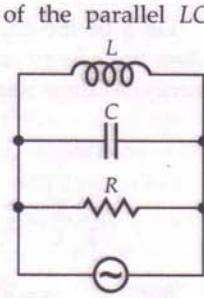


Fig. 7.83

where $|Z|$ is the modulus of the complex impedance Z. Obviously, $\frac{1}{|Z|}$ is minimum at $\omega = \omega_r$ when $\omega C = \frac{1}{\omega L}$, so that $|Z|$ is maximum and the total current amplitude is minimum. Hence, *at resonance the current in the parallel LCR-circuit is minimum.*

At resonance, $Z = R$. For total current in the circuit

$$I_{rms} = \frac{V_{rms}}{R} = \frac{230}{40} = 5.75 \text{ A}$$

The rms current in the R branch is

$$I_{rms}^{R} = \frac{V_{rms}}{R} = \frac{230}{40} = \mathbf{5.75 \text{ A}}$$

The rms current in the L branch is

$$I_{rms}^{L} = \frac{V_{rms}}{\omega_r L} = \frac{230}{50 \times 5.0} = \mathbf{0.92 \text{ A}}$$

The rms current in the C branch is

$$I_{rms}^{C} = \frac{V_{rms}}{1/\omega_r C} = V_{rms} \times \omega_r C$$

$$= 230 \times 50 \times 80 \times 10^{-6} \text{ A} = \mathbf{0.92 \text{ A}}$$

Note that the total current in the circuit is the same as that in the R branch. This is because the currents in L and C branches are 180° out of phase and add up to zero at every instant of the cycle.

7.18. *A circuit containing a 80 mH inductor and a 60 μF capacitor in series is connected to a 230 V, 50 Hz supply. The resistance of the circuit is negligible.*

(*a*) *Obtain the current amplitude and rms values.*

(*b*) *Obtain the rms values of potential drops across each element.*

(*c*) *What is the average power transferred to the inductor ?*

(*d*) *What is the average power transferred to the capacitor ?*

(*e*) *What is the total average power absorbed by the circuit ?*

Ans. Here $L = 80 \text{ mH} = 80 \times 10^{-3} \text{H}$,

$C = 60\mu \text{ F} = 60 \times 10^{-6} \text{F}$,

$V_{rms} = 230 \text{ V}$, $f = 50 \text{ Hz}$

(*a*) Reactance of the circuit

$$= \left| \omega L - \frac{1}{\omega C} \right| = \left| 2\pi f L - \frac{1}{2\pi f C} \right|$$

$$= \left| 2\pi \times 50 \times 80 \times 10^{-3} - \frac{1}{2\pi \times 50 \times 60 \times 10^{-6}} \right|$$

$$= | 25.13 - 53.05 | \, \Omega = 27.92 \, \Omega$$

$$I_{rms} = \frac{V_{rms}}{\text{Reactance}} = \frac{230}{27.92} \text{ A} = \mathbf{8.24 \text{ A}}$$

Current amplitude,

$$I_0 = \sqrt{2}\, I_{rms} = 1.414 \times 8.24 = 11.653 \simeq \mathbf{11.7 \text{ A}}.$$

(*b*) Potential drop across L is

$$V_{rms}^{L} = I_{rms} \times \omega L = 8.24 \times 25.13 = \mathbf{207 \text{ V}}$$

Potential drop across C is

$$V_{rms}^{C} = I_{rms} \times \frac{1}{\omega C} = 8.24 \times 53.05 = \mathbf{437 \text{ V}}.$$

(*c*) In an inductor, voltage leads the current by $\frac{\pi}{2}$, therefore, average power transferred to the inductor per cycle is

$$P_{av} = V_{rms}\, I_{rms} \cos \frac{\pi}{2} = 0.$$

(*d*) In a capacitor, voltage lags behind the current by $\frac{\pi}{2}$, therefore, average power transferred to the capacitor per cycle is

$$P_{av} = V_{rms}\, I_{rms} \cos\left(-\frac{\pi}{2}\right) = 0.$$

(*e*) Total average power absorbed = **0.**

7.19. *Suppose the circuit in Exercise 7.18 has a resistance of 15 Ω. Obtain the average power transferred to each element of the circuit, and the total power absorbed.*

Ans. Here $R = 15 \, \Omega$

∴ Impedance,

$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

$$= \sqrt{15^2 + \left(2\pi \times 50 \times 80 \times 10^{-3} - \frac{1}{2\pi \times 50 \times 60 \times 10^{-6}}\right)^2}$$

$$= \sqrt{225 + 779.5} = \sqrt{1004.5} \simeq 31.7 \, \Omega$$

$$\therefore \quad I_{rms} = \frac{V_{rms}}{Z} = \frac{230}{31.7} = \mathbf{7.255 \text{ A}}$$

Average power transferred to L

$$= V_{eff}\, I_{eff} \cos \frac{\pi}{2} = 0$$

Average power transferred to C

$$= V_{eff}\, I_{eff} \cos\left(-\frac{\pi}{2}\right) = 0$$

Average power transferred to R

$$= I_{rms}^2 \times R = (7.255)^2 \times 15 = \mathbf{789.5 \text{ W}}.$$

7.20. *A series LCR-circuit with L = 0.12 H, C = 480 nF, R = 23 Ω is connected to a 230 V variable frequency supply.*

(*a*) *What is the source frequency for which current amplitude is maximum ? Obtain this maximum value.*

(*b*) *What is the source frequency for which average power absorbed by the circuit is maximum ? Obtain the value of this maximum power.*

(*c*) *For which frequencies of the source is the power transferred to the circuit half the power at resonant frequency ? What is the current amplitude at these frequencies ?*

(*d*) *What is the Q-factor of the given circuit ?*

[CBSE OD 92C]

Ans. Here $L = 0.12 \text{ H}$, $C = 480 \text{ nF} = 480 \times 10^{-9} \text{F}$

$R = 23 \, \Omega$, $V_{rms} = 230 \text{ V}$

(*a*) Current amplitude I_0 is maximum at the resonant angular frequency

$$\omega_r = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{0.12 \times 480 \times 10^{-9}}} \text{ rad s}^{-1}$$

$$= \mathbf{4167 \text{ rad s}^{-1}}$$

Resonant frequency, $f_r = \frac{\omega_r}{2\pi} = \frac{4167}{2\pi} = \mathbf{663 \text{ Hz}}$

The maximum value of current amplitude is

$$I_0^{\max} = \frac{V_0}{R} = \frac{\sqrt{2}\, V_{rms}}{R} = \frac{\sqrt{2} \times 230}{23} = \mathbf{14.1 \text{ A.}}$$

(*b*) The power absorbed is maximum at the same resonant frequency (663 Hz) for which I_0 is maximum.

$$\therefore \quad P_{av}^{\max} = \frac{1}{2}(I_0^{\max})^2 R = \frac{1}{2}\frac{V_0^2}{R}$$

$$= \frac{1}{2} \cdot \frac{(\sqrt{2} \times 230)^2}{23} = \mathbf{2300 \text{ W.}}$$

(*c*) The two angular frequencies for which the power transferred to the circuit is half the power at the resonant frequency, are

$$\omega = \omega_r \pm \Delta\omega = \omega_r \pm \frac{R}{2L}$$

The corresponding frequencies will be

$$f = f_r \pm \Delta f = f_r \pm \frac{\Delta\omega}{2\pi}$$

Now $\quad \frac{\Delta\omega}{2\pi} = \frac{1}{2\pi} \times \frac{R}{2L} = \frac{1}{2\pi} \times \frac{23}{2 \times 0.12}$

$$= 15.25 \text{ Hz} \simeq \mathbf{15 \text{ Hz.}}$$

$\therefore$ Required values of f

$$= 663 \pm 15 = \mathbf{648 \text{ Hz and } 678 \text{ Hz}}$$

At these frequencies, power absorbed $= \frac{1}{2} P_{max}$. As $P \propto I^2$, the current amplitude at these half-power points

$$= \frac{1}{\sqrt{2}} I_0^{\max} = \frac{1}{\sqrt{2}} \times 14.1 = 9.97 \text{ A} \simeq \mathbf{10 \text{ A}}$$

(*d*) The Q- factor of the circuit is

$$Q = \frac{\omega_r L}{R} = \frac{4167 \times 0.12}{23} = \mathbf{21.7.}$$

7.21. *Obtain the resonant frequency and Q-factor of a series LCR-circuit with L = 3.0 H, C = 27 μF, and R = 7.4 Ω. It is desired to improve the sharpness of the resonance of the circuit by reducing its 'full width at half- maximum' by a factor of 2. Suggest a suitable way.*

Ans. Here $L = 3.0\text{H}, C = 27\mu\text{F} = 27 \times 10^{-6}$ F, $R = 7.4\,\Omega$

Resonant frequency,

$$\omega_r = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{3.0 \times 27 \times 10^{-6}}} = \mathbf{111 \text{ rad s}^{-1}}$$

Q-factor of the circuit,

$$Q = \frac{\omega_r L}{R} = \frac{111 \times 3.0}{7.4} = \mathbf{45}$$

To improve sharpness of resonance by a factor of 2, Q should be doubled. To double Q without changing ω_r, R should be reduced to half, *i.e.*, to 3.7 Ω.

7.22. *Answer the following questions :*

(a) (i) In any a.c. circuit, is the applied instantaneous voltage equal to the algebraic sum of the instantaneous voltages across the series elements of the circuit ?

(ii) Is the same true for rms voltage ?

(b) A capacitor is used in the primary circuit of an induction coil.

(c) An applied voltage signal consists of a superposition of a d.c. voltage and an a.c. voltage of high frequency. The circuit consists of an inductor and a capacitor in series. Show that the d.c. signal will appear across C and the a.c. signal across L.

(d) A choke coil in series with a lamp is connected to a d.c. line. The lamp is seen to shine brightly. Insertion of an iron core in the choke causes no change in the lamp's brightness. Predict the corresponding observations if the connection is to an a.c. line.

(e) Why is a choke coil needed in the use of fluorescent tubes with a.c. mains ? Why can we not use an ordinary resistor instead of the choke coil ?

Ans. (*a*) (*i*) Yes, because the voltage variations across each element will follow the variations of the supply voltage at all instants.

(*ii*) No, the same is not true for rms voltage, because voltages across different elements may not be in phase.

(*b*) When the primary circuit of the induction coil is broken, high voltage is induced which gets used in charging the capacitor. This avoids sparking in the circuit.

(*c*) *Inductive reactance,* $X_L = 2\pi f L$ *i.e.,* $X_L \propto f$

Capacitive reactance, $X_C = \frac{1}{2\pi fC}$ *i.e.,* $X_C = \frac{1}{f}$

For d.c., $f = 0$, reactance of L is zero and that of C is infinite, so the d.c. signal appears across C. For high frequency a.c., reactance of L is high and that of C is low. So the a.c. signal appears across L.

(*d*) For d.c., $X_L = 0$. Inductance L has no effect even if it is increased by inserting iron core. But for a.c., the lamp will shine dimly because of the impedance offered by the choke. When the iron core is inserted, impedance of the choke further increases and the lamp will dim further.

(*e*) If a fluorescent tube is connected directly across a 220 V source, it would draw large current which would damage the tube. With the use of choke coil, the voltage is reduced to an appropriate value, without wasting any power. A resistor would waste a large amount of electrical energy as heat. So an ordinary resistor cannot be used instead of a choke coil.

7.23. *A power transmission line feeds input power at 2300 V to a step down transformer having 4000 turns in its primary. What should be the number of turns in the secondary to get output power at 230 V ?* **[CBSE D97]**

Ans. Here $\mathcal{E}_1 = 2300\text{ V},\ N_1 = 4000, \mathcal{E}_2 = 230\text{ V},\ N_2 = ?$

As $$\frac{\mathcal{E}_2}{\mathcal{E}_1} = \frac{N_2}{N_1}$$

$$\therefore \quad N_2 = N_1 \cdot \frac{\mathcal{E}_1}{\mathcal{E}_2} = 4000 \times \frac{230}{2300} = \textbf{400 turns.}$$

7.24. *At a hydroelectric power plant, the water pressure head is at a height of 300 m and the water flow available is $100\ m^3s^{-1}$. If the turbine-generator efficiency is 60%, estimate the electric power available from the plant. ($g = 9.8\ ms^{-2}$).*

Ans. Hydroelectric power

$$= \frac{\text{Work}}{\text{Time}}$$

$$= \frac{\text{Force} \times \text{distance}}{\text{Time}}$$

$$= \text{Pressure} \times \text{area} \times \text{velocity}$$

$$= h\rho g \times A \times v = h\rho g \times \beta$$

where $\beta = Av$ = volume of water flowing per second across a cross-section.

Electric power available

$$= 60\% \text{ of total hydroelectric power}$$

$$= 0.6\ h\rho g\beta$$

$$= 0.6 \times 300 \times 10^3 \times 9.8 \times 100 \text{ W}$$

$$= 176.4 \times 10^6 \text{ W} \simeq \textbf{176 MW.}$$

7.25. *A small town with a demand of 800 kW of electric power at 220 V is situated 15 km away from an electric plant generating power at 440 V. The resistance of the two-wire line carrying power is $0.5\,\Omega$ per km. The town gets power from the line through a 4000 – 220 V step-down transformer at a substation in the town.*

(a) Estimate the line power loss in the form of heat.

(b) How much power must the plant supply, assuming there is negligible power loss due to leakage ?

(c) Characterise the step-up transformer at the plant.

[CBSE Sample Paper 03]

Ans. Line resistance

= Length of two-wire line × Resistance per unit length

$$= 2 \times 15 \text{ km} \times 0.5\,\Omega\text{ km}^{-1} = 15\,\Omega$$

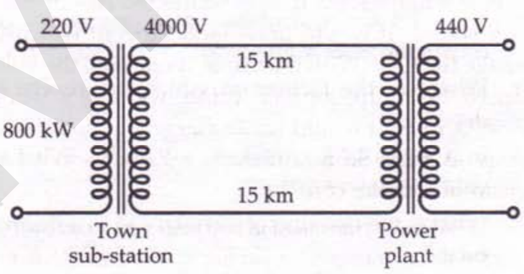


Fig. 7.84

Voltage at which power is sent through the line

$$= 4000 \text{ V}$$

Power supplied to town sub-station

$$= 800 \text{ kW} = 800 \times 10^3 \text{ W}$$

$\therefore$ rms value of current in the line

$$= \frac{\text{Power}}{\text{Voltage}} = \frac{800 \times 10^3}{4000} \text{ A} = 200 \text{ A}$$

(*a*) Line power loss

$$= I^2 R = (200)^2 \times 15 \text{ W} = \textbf{600 kW.}$$

(*b*) Power supplied by the plant

= Power received at sub-station + line power loss

$$= 800 + 600 = \textbf{1400 kW.}$$

(*c*) Voltage drop on the line

$$= IR = 200 \times 15 = 3000 \text{ V}$$

Voltage output of the step-up transformer at the plant

$$= 4000 + 3000 = 7000 \text{ V}$$

Hence the step-up transformer at the plant is **440 – 7000 V.**

7.26. *Do the same exercise as above with the replacement of the earlier transformer by a 40,000 – 220 V step-down transformer (Neglect, as before, leakage losses though this may not be a good assumption any longer because of the very high voltage transmission involved). Hence, explain why high voltage transmission is preferred.*

Ans. The rms current in the two-wire line

$$= \frac{800 \times 10^3 \text{ W}}{40{,}000 \text{ V}} = 20 \text{ A}$$

(*a*) Line power loss

$$= I^2 R = (20)^2 \times 15 = 6000 \text{ W} = \textbf{6 kW.}$$

(*b*) Power supplied by the plant

$$= 800 + 6 = \textbf{806 kW.}$$

(*c*) ge drop on the line

$$= IR = 20 \times 15 = \textbf{300 V.}$$

Voltage output of the step-up transformer at the plant

$$= 40{,}000 + 300 = 40{,}300 \text{ V}$$

$\therefore$ The step-up transformer at the plant is **440 V – 40,300 V.**

Power loss in exercise 7.25

$$= \frac{600}{1400} \times 100 = 43\%$$

Power loss in exercise 7.26

$$= \frac{6}{806} \times 100 = 0.74\%$$

Thus the percentage power loss is greatly reduced by high voltage transmission. At high voltage transmission, a small current flows and hence power loss is less ($P \propto I^2$).

Text Based Exercises

TYPE A : VERY SHORT ANSWER QUESTIONS (1 mark each)

1. What is an alternating current ? Write an expression for its instantaneous value.
2. What is the frequency of direct current ? [Himachal 99C]
3. Define the mean value of an alternating current. [Punjab 01]
4. Define the root mean square value of an alternating current. [ISCE 95 ; Punjab 01]
5. What is the relation between peak value and the root mean square value of an alternating emf ? [Haryana 01]
6. The instantaneous current from an a.c. source is $I = 5 \sin 314\, t$. What is the rms value of the current? [CBSE OD 96, 2000]
7. The instantaneous voltage from an a.c. source is given by $\mathcal{E} = 300 \sin 314\, t$. What is the rms voltage of the source ? [CBSE OD 2000]
8. The emf of an a.c. source is given by the expression :
$$\mathcal{E} = 300 \sin 314\, t$$
Write the values of peak voltage and frequency of the source. [CBSE D 93C]
9. What is peak value of voltage for 220 V a.c. ?
10. The divisions marked on the scale of an a.c. ammeter are not equally spaced. Why ?
11. What is form-factor ?
12. Define inductive reactance. Give its SI unit.
13. Define capacitive reactance. Give its SI unit. [CBSE D 15]
14. How does the capacitive reactance depend on frequency of a.c. ?
15. How does inductive reactance depend on frequency of a.c. ?
16. What is the SI unit of $1/\omega C$?
17. What is the reactance of a capacitor of capacitance C at f hertz ? [ISCE 97]
18. What is the phase relationship between current and voltage in an inductor ? [Haryana 01, 02]
19. What is phase relationship between current and emf in an a.c. circuit containing a capacitor only ?
20. Sketch a graph showing the variation of inductive reactance with frequency of the applied voltage. [Haryana 02 ; CBSE OD 02]
21. Sketch a graph showing the variation of the reactance of a capacitor with frequency of the applied voltage. [CBSE OD 01, 15C]
22. What is the phase difference between the voltage across L and C in a series LCR-circuit connected to an a.c. source ? [CBSE D 98C]
23. Give the phase difference between the applied a.c. voltage and the current in an LCR circuit at resonance. [CBSE OD 97]
24. What is the capacitive reactance of a capacitor used in a circuit having d.c. emf ? Give reason. [Pb 92]
25. What is impedance ? [Punjab 96C]
26. When does a series LCR-circuit have minimum impedance ?
27. A series LCR-circuit with $L = 0.12\,\text{H}$, $C = 4.8 \times 10^{-7}\,\text{F}$, $R = 23\,\Omega$ is connected to a variable frequency supply. At what frequency is the current maximum ? [CBSE OD 90]
28. What is meant by admittance of an a.c. circuit ?
29. Define power factor. [Haryana 01 ; Punjab 01]
30. What are the maximum and minimum values of power factor of a.c. circuit ? [Punjab 99C]
31. What is the power consumed (*i*) in purely inductive (*ii*) purely capacitive a.c. circuits ? [CBSE OD 92]
32. What is the power factor of an LCR series circuit at resonance ? [ISCE 94 ; CBSE D 99]
33. What is wattless current ? [ISCE 2000 ; CBSE D 11]
34. A series LCR-circuit is connected to an a.c. source. In which of its part power dissipation occurs–L, C or R ?
35. What is quality factor (Q) in an a.c. circuit ? [CBSE OD 92]
36. What does quality factor (Q) signify in a series LCR-circuit ?
37. Give the expression for power factor in an LR-circuit in terms of the resistance R and the inductive reactance X_L. [ISCE 02]
38. In an a.c. circuit, $R = 4\,\Omega$, $Z = 5\,\Omega$, $V_{rms} = 200$ V and $I_{rms} = 1.5$ A. Calculate the average power consumed over a full cycle. [ISCE 03]
39. What are the factors on which the power factor depends ? [Haryana 98]
40. In a series LCR circuit, $V_L = V_C \neq V_R$. What is the value of power factor ? [CBSE OD 15]
41. What is the function of a choke coil in a fluorescent tube ? [ISCE 96 ; CBSE D 14]
42. How can we improve the Q-factor of a series resonant circuit ?

43. What are (*i*) dimensions and (*ii*) units of product RC ?
44. Is the direction of current during the discharging of capacitor same as that during the charging process ?
45. What is the natural frequency of LC-circuit ? What is the reactance of this circuit at this frequency ?
46. What are the dimensions of $\sqrt{LC}$?
47. In a pure LC-circuit, what is the energy stored when peak current is I_0 ?
48. Where does the energy reside in a charged capacitor ?
49. Where does the energy reside in an inductor when a current I_0 is established in it ?
50. State the principle of a.c. generator. [Haryana 95]
51. How can an a.c. generator be converted into d.c. generator ? [Punjab 04]
52. State the principle of a transformer. [Haryana 97C]
53. An a.c. voltage of 200 V is applied to the primary of a transformer and voltage of 2000 V is obtained from the secondary coil. Calculate the ratio of currents in the primary and secondary coils. [CBSE Sample Paper 90]
54. Give two reasons for power loss in a transformer.
55. Which device will you use to step up a.c. voltage ? Can we use the same device to set up d.c. voltage ? [CBSE D 93C]
56. What is copper loss in a transformer ? [Punjab 99]
57. What is iron loss in a transformer ? [Punjab 2000]
58. How can iron loss in a transformer be reduced ? [Punjab 98C, 99]
59. In a series LCR-circuit, the voltage across an inductor, capacitor and resistor are 20 V, 20 V and 40 V respectively. What is the phase difference between the applied voltage and the current in the circuit ? [CBSE OD 05]
60. The power factor of an a.c. circuit is 0.5. What will be the phase difference between voltage and current in this circuit ? [CBSE D 05 ; F 15]
61. Give expression for the average value of the a.c. voltage $V = V_0 \sin \omega t$ over the time interval $t = 0$ and $t = \frac{\pi}{\omega}$. [CBSE Sample Paper 08]
62. The instantaneous current and voltage of an a.c. circuit are given by
$$i = 10 \sin 314t \text{ A and } v = 50 \sin 314t \text{ V}.$$
What is the power dissipation in the circuit ? [CBSE OD 08]
63. The instantaneous current and voltage of an a.c. circuit are given by
$$i = 10 \sin 314t \text{ A and } v = 50 \sin \left(314t + \frac{\pi}{2}\right) \text{ V}.$$
What is the power dissipation in the circuit ? [CBSE OD 08]
64. An electrical element X, when connected to an alternating voltage source, has the current through it leading the voltage by $\pi/2$ radian. Identify X and write an expression for its reactance. [CBSE SP 08]
65. If the average power, supplied by an a.c. source, to a given circuit element, over a complete cycle, is found to be zero, what can be the possible nature of this element ? [CBSE D 09C]
66. State the steady value of the reading of the ammeter in the circuit shown in Fig. 7.80.

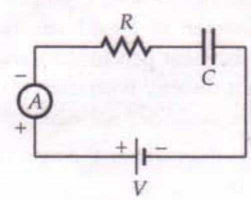


Fig. 7.85

67. State which of the two, the capacitor or an inductor, tends to become a SHORT when the frequency of the applied alternating voltage has a very high value. [CBSE SP 15]
68. Why is the use of a.c. voltage preferred over d.c. voltage ? Give two reasons. [CBSE OD 14]

Answers

1. An alternating current is that current which changes continuously in magnitude and periodically in direction. The value of a.c. at any instant is given by
$$I = I_0 \sin \omega t = I_0 \sin 2\pi ft$$
where I_0 is the peak value of current and
$$\omega = 2\pi f = 2\pi / T,$$
is the angular frequency of a.c.
2. The frequency of d.c. is zero.
3. **Average value of a.c.** It is that value of direct current which sends the same charge in a circuit in the same time as is sent by the given alternating current in its half time period.
$$I_{av} = \frac{2}{\pi} I_0 = 0.637\, I_0$$

4. **RMS value of a.c.** It is that value of a direct current which produces the same heating effect in a given resistor as is produced by the given alternating current when passed for the same time.

$$I_{rms} = \frac{I_0}{\sqrt{2}} = 0.707\, I_0,$$

where I_0 is the peak value of a.c.

5. $\mathcal{E}_{rms} = \frac{1}{\sqrt{2}} \mathcal{E}_0$.

6. Here $I_0 = 5$ amperes

$$\therefore \quad I_{rms} = \frac{I_0}{\sqrt{2}} = 0.707 \times 5 = \mathbf{3.535\ A.}$$

7. Here $\mathcal{E}_0 = 300$ volts

$$\therefore \quad \mathcal{E}_{rms} = \frac{\mathcal{E}_0}{\sqrt{2}} = 0.707 \times 300 = \mathbf{212.1\ V.}$$

8. Comparing the given expression : $\mathcal{E} = 300 \sin 314\, t$ with $\mathcal{E} = \mathcal{E}_0 \sin 2\pi\, ft$, we find that,

Peak voltage,

$$\mathcal{E}_0 = \mathbf{300\ V} \quad \text{and} \quad 2\pi f = 314$$

$$\therefore \text{Frequency,} \quad f = \frac{314}{2\pi} = \frac{314}{2 \times 3.14} = \mathbf{50\ Hz.}$$

9. $\mathcal{E}_0 = \sqrt{2}\mathcal{E}_{rms} = \sqrt{2} \times 220 = \mathbf{311\ V.}$

10. An a.c. ammeter is based on heating effect of current. As the heat produced varies as the square of current (not directly with current), so the divisions marked on the scale are not equally spaced.

11. Form factor is the ratio of the rms value of a.c. to its average value.

12. The inductive reactance of an inductor is the effective resistance offered by it to the flow of current through it.

Inductive reactance, $X_L = \omega L = 2\pi f L$

SI unit of inductive reactance is *ohm* (Ω)

13. The capacitive reactance of a capacitor is the effective resistance offered by it to the flow of current through it.

$$\text{Capacitive reactance, } X_C = \frac{1}{\omega C} = \frac{1}{2\pi\, f\, C}$$

SI unit of capacitive reactance is *ohm* (Ω).

14. Capacitive reactance varies inversely with the frequency of a.c. *i.e.*, $X_C \propto \frac{1}{f}$

15. Inductive reactance varies directly with the frequency of a.c. *i.e.*, $X_L \propto f$.

16. The SI unit of $1/\omega C$ is ohm (Ω).

17. Capacitive reactance, $X_C = \frac{1}{2\pi\, f\, C}$.

18. In an inductor, the current lags behind the voltage by a phase angle of $\pi/2$ rad.

19. In a capacitive a.c. circuit, current leads the emf in phase by $\pi/2$ radian.

20. See Fig. 7.58(*a*) on page 7.57.

21. See Fig. 7.58(*b*) on page 7.57.

22. 180° or π radian.

23. Zero.

24. For d.c., $f = 0$, so $X_C = \frac{1}{2\pi\, f\, C} = \infty$.

25. The total opposition to the flow of current due to resistance R as well as reactance X in a circuit is called impedance. It is given by $Z = \sqrt{R^2 + X^2}$

26. At resonance, when $X_L = X_C$.

27. The current will be maximum at the resonant frequency,

$$f_r = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{0.12 \times 4.8 \times 10^{-7}}}$$

$$= \frac{1}{2\pi\sqrt{5.76 \times 10^{-8}}} = \frac{10^4}{2 \times 3.14 \times 2.4} \simeq \mathbf{663\ Hz.}$$

28. The reciprocal of the impedance of an a.c. circuit is called its admittance.

29. The power factor is defined as the ratio of true power to the apparent power of an a.c. circuit. It is equal to the cosine of the phase angle between current and voltage in the a.c. circuit. It is given by

$$\cos\phi = \frac{\text{True power}}{\text{Apparent power}} = \frac{P_{av}}{V_{rms}\, I_{rms}}$$

For a series LCR-circuit, power factor is

$$\cos\phi = \frac{R}{Z} = \frac{R}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

30. Maximum value of power factor = **1**.

Minimum value of power factor = **0**.

31. Power consumed is zero in both cases.

32. At resonance, $Z = R$.

$$\therefore \text{Power factor,} \quad \cos\phi = \frac{R}{Z} = \frac{R}{R} = 1.$$

33. The current in an a.c. circuit is wattless if the average power consumed in the circuit is zero.

34. Power dissipation occurs only in resistance R.

35. Refer to point 18 of Glimpses.

36. The quality factor Q signifies the sharpness of current peak in the resonance condition of the LCR-circuit.

37. Power factor, $\cos\phi = \frac{R}{Z} = \frac{R}{\sqrt{R^2 + X_L^2}}$.

38. $P_{av} = V_{rms}\, I_{rms} \cos\phi = V_{rms}\, I_{rms} \cdot \frac{R}{Z}$

$= 200 \times 1.5 \times \frac{4}{5} = \mathbf{240\ W}.$

39. Power factor of a circuit depends on the values of L, and R, and the frequency of a.c. source.

40. Power factor, $\cos\phi = \mathbf{1}$.

41. A choke coil reduces current without wasting electrical energy in the form of heat.

42. By decreasing R or C or by increasing L.

43. (*i*) $M^0L^0T^1$ (*ii*) Second.

44. No, the direction of current during discharging is opposite to that during charging of the capacitor.

45. The natural frequency of LC-circuit is $f = \frac{1}{2\pi\sqrt{LC}}$.

At this frequency, $X_L = X_C$, so the reactance of the circuit is zero.

46. $[M^0L^0T^1]$.

47. Total energy stored $= \frac{1}{2} L I_0^2$.

48. The energy resides in the electric field between the plates of a charged capacitor.

49. The energy resides in the magnetic field of the inductor coil.

50. An a.c. generator is based on the principle of electromagnetic induction. When a closed coil is rotated in a uniform magnetic field with its axis perpendicular to the field, the magnetic flux linked with the coil changes and an induced emf and hence an induced current is set up in the coil.

51. By replacing slip rings by split ring commutator.

52. A transformer works on the principle of mutual induction *i.e.*, when a changing current is passed through one coil, an induced emf is set up in the neighbouring coil.

53. Here $\mathcal{E}_1 = 200$ V, $\mathcal{E}_2 = 2000$ V

Ratio of currents, $\frac{I_1}{I_2} = \frac{\mathcal{E}_2}{\mathcal{E}_1} = \frac{2000}{200} = \mathbf{10 : 1}.$

54. (*i*) Magnetic flux leakage (*ii*) Hysteresis loss.

55. (*i*) A transformer is used to step up a.c. voltage.

(*ii*) No, transformer cannot be used to step up d.c. voltage.

56. Due to the resistance of primary and secondary windings of a transformer, some electrical energy is wasted as heat. This energy loss is called copper loss.

57. Due to the production of eddy currents in the iron core of a transformer, some electrical energy is wasted as heat. This energy loss is called iron loss.

58. Iron loss can be reduced by using laminated core in the transformer.

59. $\tan\phi = \frac{V_L - V_C}{V_R} = \frac{20 - 20}{40} = 0$

$\therefore$ Phase difference, $\phi = \mathbf{0°}$.

60. Power factor $= \cos\phi = 0.5$

$\therefore$ Phase difference, $\phi = \mathbf{60°}$.

61. $V_{av} = \frac{2}{\pi} V_0 = 0.637\, V_0$.

62. $P_{av} = \frac{v_0 i_0}{2} \cos\phi = \frac{50 \times 10}{2} \cos 0° = \mathbf{250\ W}$

63. Phase difference between v and $i = \pi / 2$ rad.

$\therefore\ P_{av} = \frac{v_0 i_0}{2} \cos\phi = \frac{50 \times 10}{2} \cos\frac{\pi}{2} = 0$

64. The element X is a capacitor and its reactance is given by $X_C = \frac{1}{2\pi f C}$

65. The element may be a pure inductor or a capacitor.

66. Zero, because a capacitor blocks dc.

67. Capacitor, as $f \to \infty$, $X_C = \frac{1}{2\pi f C} \to 0$.

68. (*i*) a.c. voltages can be easily stepped up or stepped down as per requirement by using transformers.

(*ii*) a.c. voltages can be transmitted over large distances without any appreciable loss of energy.

TYPE B : SHORT ANSWER QUESTIONS (2 or 3 marks each)

1. Distinguish between alternating current and direct current.
2. Distinguish between average value and rms value of an alternating current. [CBSE D 97]
3. Prove mathematically that the average value of alternating current over one complete cycle is zero.
4. Define peak value and root mean square value of an alternating current. Derive an expression for the root mean square value of alternating current.
5. What is meant by the root mean square or effective value of alternating current ? Derive a relation between it and its peak value. [CBSE OD 11]
6. Define virtual emf and derive the relation between virtual emf and maximum emf in a.c. [Haryana 92]
7. Prove that the voltage and current always vary in the same phase in an a.c. circuit containing resistance only. Show this phase relationship graphically.

8. A sinusoidal emf is applied to a circuit containing an inductor only. Show that the current lags behind the voltage by $\pi/2$. Show this phase relationship graphically. [CBSE OD 11]

9. An a.c. voltage, $\mathcal{E} = \mathcal{E}_0 \sin \omega t$ is applied across an inductance L. Obtain an expression for the current I. Show the phase relationship between current and voltage in a phasor diagram. [CBSE D 03 ; Pb 01]

10. A sinusoidal emf is applied to a circuit containing a capacitor only. Show that current leads the voltage by $\pi/2$. [CBSE OD 95C]

11. Derive the expression for the reactance of a capacitor C, when connected across an a.c. source. Give its units.

12. Derive an expression for the impedance of an a.c. circuit consisting of an inductor and a resistor. [CBSE D 08]

13. A source of ac voltage $V = V_0 \sin \omega t$ is connected to a series combination of a resistor 'R' and a capacitor 'C'. Draw the phasor diagram and use it to obtain the expression for (*i*) impedance of the circuit and (*ii*) phase angle. [CBSE OD 15C]

14. Obtain the relation $I = I_0 \sin(\omega t + \pi/2)$ and $X_C = 1/\omega C$ for a pure capacitor across which an a.c. emf of $\mathcal{E} = \mathcal{E}_0 \sin \omega t$ is applied as shown in Fig. 7.86. Draw a phasor diagram showing emf $\mathcal{E}$, current I and their phase difference ϕ. [ISCE 03]

15. An alternating emf $\mathcal{E} = \mathcal{E}_0 \sin \omega t$ is applied to a capacitor C, as shown in above Fig. 7.86.

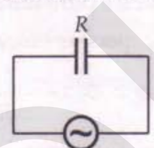


Fig. 7.86

(*i*) Sketch a graph showing variation of voltage and current in the circuit with time.

(*ii*) What is the reactance of the capacitance ? [ISCE 92]

16. (*a*) For a given ac, $i = i_m \sin \omega t$, show that the average power dissipated in a resistor in a resistor R over a complete cycle is $\frac{1}{2} i_m^2 R$.

(*b*) A light bulb is rated at 120 W, for a 240 V ac supply. Calculate the resistance of the bulb. [CBSE OD 13]

17. Prove that an ideal inductor does not dissipate power in an a.c. circuit. [CBSE D 08 ; OD 98]

18. When an ideal capacitor is connected to an a.c. source, show that the average power supplied by the source over a complete cycle is zero. [CBSE D 08, 13C]

19. Define self-inductance of a coil. Write its SI unit. Show that the energy needed to build up a current I, in an inductor of self-inductance L, equals $\frac{1}{2} LI^2$. [CBSE D 09C, 12 ; OD 15]

20. A voltage $V = V_0 \sin \omega t$ is applied to a series LCR circuit. Derive the expression for the average power dissipated over a cycle. Under what condition is (*i*) no power dissipated even though the current flows through the circuit, (*ii*) maximum power dissipated in the circuit ? [CBSE OD 14]

21. A series LCR circuit is connected to an a.c. source of variable frequency. Draw a suitable phasor diagram to deduce the expressions for the amplitude of the current and phase angle. [CBSE D 14C]

22. Distinguish between reactance and impedance. [CBSE D 97 ; OD 99]

23. For a given a.c. circuit, distinguish between resistance, reactance and impedance. [CBSE D 2000]

24. Show that energy is conserved in LC-oscillations.

25. Give a mechanical analogy for LC-oscillations.

26. With the help of a labelled diagram, explain the working principle of a step-up transformer. [CBSE 98C, 02]

27. What are the various energy losses in a transformer ? How can they be reduced ? [Himachal 02 ; Punjab 2000, 02]

28. Draw a labelled diagram of an a.c. generator. Write the principle on which it works. [CBSE D 98C, 02]

29. Explain, with the help of diagram, the principle and working of an a.c. generator. Write the expression for the e.m.f. generated in the coil in terms of its speed of rotation. [CBSE D 05]

30. State the underlying principle of a transformer. How is the large scale transmission of electric energy over long distances done with the use of transformers ? [CBSE OD 12]

31. Write some advantages and disadvantages of a.c. over d.c.

32. What is a choke ? Explain its action in a.c. circuits. Why is it preferred to resistance in a.c. circuits ? [CBSE OD 92C]

33. Derive a relation between true power and virtual power. How will you differentiate between true power and apparent power ? [Punjab 02, 04]

34. A series LCR circuit is connected to an ac source. Using the phasor diagram, derive the expression for the impedance of the circuit. Plot a graph to show the variation of current with frequency of the source, explaining the nature of its variation. [CBSE OD 12]

Answers

1. Refer to points 1 and 2 of Glimpses.
2. Refer to points 3 and 4 of Glimpses.
3. Refer answer to Q. 2 on page 7.2.
4. Refer answer to Q. 5 on page 7.3.
5. Refer answer to Q. 5 on page 7.3.
6. Refer answer to Q. 6 on page 7.3.
7. Refer answer to Q. 7 on page 7.7.
8. Refer answer to Q. 8 on page 7.7.
9. Refer answer to Q. 8 on page 7.7.
10. Refer answer to Q. 10 on page 7.9.
11. Refer answer to Q. 10 on page 7.9.
12. Refer answer to Q. 11 on page 7.12.
13. Refer answer to Q. 12 on page 7.17.
14. Refer answer to Q. 10 on page 7.9.
15. Refer answer to Q. 10 on page 7.9.
16. (*a*) Refer answer to Q. 23 on page 7.32.

    (*b*) $R = \frac{V^2}{P} = \frac{240 \times 240}{120} = \mathbf{480\,\Omega}$
17. Refer answer to Q. 25 on page 7.32.
18. Refer answer to Q. 27 on page 7.33.
19. Refer answer to Q. 24 on page 7.32.
20. Refer answer to Q. 20 on page 7.30.

    (*i*) No power is dissipated when $R = 0$ or $\phi = 90°$.

    (*ii*) Maximum power is dissipated at resonance, *i.e.*, when $X_L = X_C$, $\phi = 0$ and $\cos\phi = 1$

    Then $P_{max} = \frac{V_0 I_0}{2}$
21. Refer answer to Q. 13 on page 7.21.
22. Refer to solution of Problem 2 on page 7.56.
23. Refer to solution of Problem 2 on page 7.56.
24. Refer answer to Q. 30 on page 7.40.
25. Refer answer to Q. 31 on page 7.41.
26. Refer answer to Q. 32 on page 7.43.
27. Refer answer to Q. 32 on page 7.43.
28. Refer answer to Q. 35 on page 7.48.
29. Refer answer to Q. 35 on page 7.48.
30. Refer answer to Q. 34 on page 7.45.
31. Refer answer to Q. 36 on page 7.50.
32. Refer answer to Q. 19 on page 7.30.
33. Refer answer to Q. 21 on page 7.31.
34. Refer answer to Q. 13 on page 7.21. For the graph showing the variation of current with the frequency of a.c. source, see Fig. 7.70. The current is maximum at the resonant frequency, $f_r = \frac{1}{2\pi\sqrt{LC}}$ and it decreases both for lower and higher frequencies.

TYPE C : LONG ANSWER QUESTIONS (5 marks each)

1. (*a*) Derive the relationship between the peak and the rms value of current in an a.c. circuit.

   (*b*) Show that in an a.c. circuit containing a pure inductor, the voltage is ahead of current by $\pi/2$ in phase. [CBSE OD 11]
2. An a.c. voltage $V = V_m \sin\omega t$ is applied across an inductor of inductance *L*. Apply Kirchhoff's loop rule to obtain expressions for (*i*) the current flowing in the circuit (*ii*) the inductive reactance *L*.

   Hence find the instantaneous power P_i supplied to the inductor. Show graphically the variation of P_i with ωt. [CBSE SP 15]
3. An a.c. source generating a voltage $v = v_m \sin\omega t$ is connected to a capacitor of capacitance *C*. Find the expression for the current *i*, flowing through it. Plot a graph of *v* and *i* versus ωt to show that the current is $\pi/2$ ahead of the voltage. What is capacitive reactance. [CBSE OD 03, 08]
4. Derive an expression for the impedance of an a.c. circuit with an inductor *L* and a resistor *R* in series. Also obtain the expression for average power in this circuit. [Punjab 01]
5. Distinguish between the terms resistance and impedance of an a.c. circuit. A capacitor *C* and a resistor *R* are connected in series in an a.c. circuit. Deduce, by drawing phasor diagram, a mathematical expression for the impedance of this circuit. How will this impedance be affected when the frequency of the applied signal is decreased and why ? [CBSE Sample Paper 03]
6. A series *LCR* circuit is connected to an a.c. source having voltage $V = V_m \sin\omega t$. Derive the expression for the instantaneous current *I* and its phase relationship to the applied voltage.

   Obtain the condition for resonance to occur. Define '*power factor*'. State the conditions under which it is (*i*) maximum and (*ii*) minimum. [CBSE D 10, F 13]
7. (*a*) In a series *LCR* a.c. circuit, is the applied instantaneous voltage equal to the algebraic sum of the instantaneous voltages across the series elements of the circuit ? Is the same true for r.m.s. voltages ?

(*b*) Prove that in a series *LCR* circuit, the power dissipated depends not only on the voltage and current but also on the cosine of the phase angle ϕ between these two. [CBSE S.P. 11]

8. Derive an expression for the impedance of a series *LCR* circuit connected to an a.c. supply of variable frequency.

   Plot a graph showing variation of current with the frequency of the applied voltage.

   Explain briefly how the phenomenon of resonance in the circuit can be used in tuning mechanism of a radio or a TV set. [CBSE D 11]

9. An a.c. voltage, $\mathcal{E} = \mathcal{E}_0 \sin \omega t$, is applied across a series combination of an inductor L, capacitor C and a resistor R. Use the phasor diagram to obtain expressions for the (*a*) impedance of the circuit, and (*b*) phase angle between the applied voltage and the resulting current in the circuit. Hence show that the current

   (*i*) leads the voltage when $\omega < \dfrac{1}{\sqrt{LC}}$

   (*ii*) is in phase with voltage when $\omega = \dfrac{1}{\sqrt{LC}}$

   (*iii*) lags the voltage when $\omega > \dfrac{1}{\sqrt{LC}}$

   What is the net impedance of the circuit when $\omega = \dfrac{1}{\sqrt{LC}}$? [CBSE D 07C, OD 13C]

10. An a.c. source of voltage $v = v_m \sin \omega t$ is connected, one-by-one, to three circuit elements X, Y and Z. It is observed that the current flowing in them,

    (*i*) is in phase with the applied voltage for element X.

    (*ii*) lags the applied voltage, in phase by $\pi/2$ for element Y.

    (*iii*) leads the applied voltage, in phase, by $\pi/2$ for element Z.

    Identify the three circuit elements.

    Find an expression for the (*a*) current flowing in the circuit, (*b*) net impedance of the circuit, when the same a.c. source is connected across a series combination of the elements X, Y and Z.

    If the frequency of the applied voltage is varied, set up the condition of the frequency when the current amplitude in the circuit is maximum. Write the expression for this current amplitude. [CBSE 08C]

11. What is meant by *LC*-oscillations ? Discuss qualitatively, how these oscillations are produced.

12. In a series *LCR* circuit connected to an ac source of variable frequency and voltage $v = v_m \sin \omega t$, draw a plot showing the variation of current (I) with angular frequency (ω) for two different values of resistance R_1 and $R_2 (R_1 > R_2)$. Write the condition under which the phenomenon of resonance occurs. For which value of the resistance out of the two curves, a sharper resonance is produced ? Define *Q*-factor of the circuit and give its significance. [CBSE D 13, 14C]

    Or

    Draw a plot showing the variation of the current I as a function of angular frequency ω of the applied ac source for the two cases of a series combination of :

    (*i*) Inductance L_1, capacitance C_1 and resistance R_1 and

    (*ii*) Inductance L_2, capacitance C_2 and resistance R_2, where $R_2 > R_1$. Write the relation between L_1, C_1 and L_2, C_2 at resonance. Which one, of the two, would be better suited for fine tuning in a receiver set ? Give reason. [CBSE F 13]

13. (*a*) With the help of a labelled diagram, describe briefly the underlying principle and working of a step-up transformer.

    (*b*) Write any two sources of energy loss in a transformer.

    (*c*) A step-up transformer converts a low input voltage into a high output voltage. Does it violate law of conservation of energy ? Explain. [CBSE D 09, 11, OD 13C]

14. (*a*) Draw a schematic arrangement for winding of primary and secondary coils in a transformer when the two coils are wound on top of each other.

    (*b*) State the underlying principle of a transformer and obtain the expression for the ratio of secondary to primary voltage in terms of the : (*i*) number of secondary and primary windings and (*ii*) primary and secondary currents.

    (*c*) Write the main assumption involved in deriving the above relations.

    (*d*) How is the transformer used in large scale transmission and distribution of electrical energy over long distances ? [CBSE F 09 ; OD 08 10, 14C]

15. Explain briefly, with the help of a labelled diagram, the basic principle of the working of an a.c. generator.

    In an a.c. generator, coil of N turns and area A is rotated at ν revolutions per second in a uniform magnetic field B. Derive an expression for the instantaneous value of the emf induced in coil. Why is the emf maximum when the plane of the armature is parallel to the magnetic field ?

    What is the source of energy generation in this device ? [CBSE OD 08 11 ; F 09]

16. (*a*) Draw a schematic sketch of an ac generator describing its basic elements. State briefly its working principle. Show a plot of variation of : (*i*) Magnetic flux and (*ii*) Alternating emf versus time generated by a loop of wire rotating in a magnetic field.

(*b*) Can the current produced by an a.c. generator be measured with a moving coil galvanometer ? Give reasons for your answer. [CBSE D 14]

Answers

1. (*a*) Refer answer to Q. 5 on page 7.3.
   (*b*) Refer answer to Q. 8 on page 7.7
2. Refer answer to Q. 8 on page 7.7 and solution of Problem 19(*ii*) on page 7.59.
3. Refer answer to Q. 10 on page 7.9.
4. Refer answer to Q. 11 on page 7.12.

$$P_{av} = \mathcal{E}_{\text{rms}} \cdot I_{\text{rms}} \frac{R}{Z}$$

$$= \mathcal{E}_{\text{rms}} \cdot I_{\text{rms}} \frac{R}{\sqrt{R^2 + \omega^2 L^2}}.$$

5. Refer to solution of Problem 2 on page 7.56 and refer answer to Q. 12 on page 7.17.

   Impedance of *RC*-circuit,

$$Z = \sqrt{R^2 + \frac{1}{4\pi^2 f^2 C^2}}$$

   As the frequency of the applied signal decreases, the impedance Z increases.

6. Refer answer to Q. 13 on page 7.21 and Q. 21 on page 7.31.
7. (*a*) (*i*) Yes, $V = V_L + V_C + V_R$ for instantaneous values because the voltage variations across each element will follow the voltage variations of the supply voltage at all instants.

   (*ii*) No, the same is not true for rms voltage, because voltages across different elements may not be in the same phase.

$$V_{\text{rms}} = \sqrt{(V_L - V_C)^2 + V_R^2}$$

   (*b*) Refer answer to Q. 20 on page 7.30.

8. Refer answer to Q. 13 on page 7.21. For graph between I and f, see Fig. 7.70 on page 7.62.

   The tuning circuit of a radio or TV is an example of *LCR* resonant circuit. Signals are transmitted by different stations at different frequencies. The antenna receives these signals and drives maximum current through the tuning circuit for that signal which corresponds to resonant frequency and so the signal from the desired station gets tuned in.

9. Refer answer to Q. 13 on page 7.21 and Q. 15 on page 7.22.
10. X is a resistor, Y is an inductor and Z is a capacitor. Refer answer to Q. 13 on page 7.21 and Q. 15 on page 7.22.
11. Refer answer to Q. 28 on page 7.39.
12. The variation of current I with angular frequency ω of *LCR*-circuit for two resistances R_1 and R_2 ($R_1 > R_2$) is shown in Fig. 7.87.

Fig. 7.87

When $X_L = X_C$ or $V_L = V_C$, the *LCR*-circuit is in resonance condition.

The resonance peak is sharp for smaller resistance R_2.

The *Q*-factor of a series resonant circuit is the ratio of the resonant frequency to the difference in two frequencies taken on both sides of the resonant frequency such that at each frequency, the current amplitude becomes $\frac{1}{\sqrt{2}}$ times the value at resonant frequency.

$$Q = \frac{\omega_r}{\omega_2 - \omega_1} = \frac{\omega_r}{2\Delta\omega}$$

$$= \frac{\text{Resonant frequency}}{\text{Bandwidth}}$$

$$= \frac{\omega_0 L}{R} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

The *Q*-factor signifies the sharpness of the peak in the resonance condition of the *LCR*-circuit. For large *Q*-factor, resonance will be sharper and consequently the circuit will be more selective.

Resonant frequency,

$$\omega_r = \frac{1}{\sqrt{L_1C_1}} = \frac{1}{\sqrt{L_2C_2}}$$

$$\Rightarrow \quad L_1C_1 = L_2C_2$$

13. (*a*) Refer answer to Q. 32 on page 7.43.
(*b*) Refer answer to Q. 32 on page 7.43.
(*c*) A step-up transformer increases low voltage into high voltage but decreases high current into low current in the same ratio. There is no power change. Hence it does not violate the principle of conservation of energy.

14. Refer answer to Q. 32 on page 7.43.

15. Refer answer to Q. 35 on page 7.48. When the plane of the armature is parallel to the magnetic field, the rate of change of flux is maximum,

$$\mathcal{E} = \mathcal{E}_0 \sin\frac{\pi}{2} = \mathcal{E}_0$$

16. (*a*) See Fig. 7.53 on page 7.49. An a.c. generator works on the principle of electromagnetic induction. When a closed coil is rotated continuously in a uniform magnetic field with its axis perpendicular to the field, the magnetic flux linked with the coil changes due to the change in the effective area of the coil. This results in the production of induced emf and hence a current in the coil.

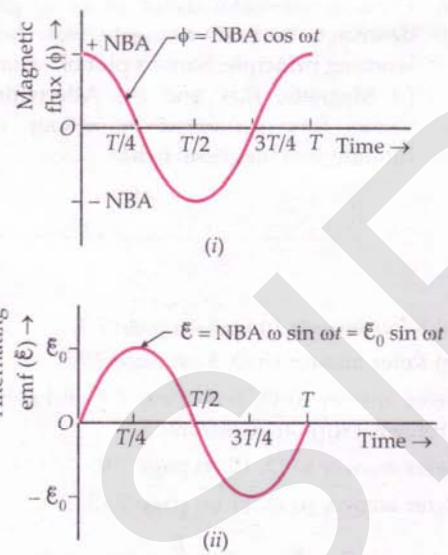


Fig. 7.88

(*b*) The current produced by an a.c. generator cannot be measured with a moving coil galvanometer because the average value of a.c. over a complete cycle is zero. The source of energy is the device used to provide mechanical energy for the rotation of the coil.

TYPE D : VALUE BASED QUESTIONS (4 marks each)

1. One day Gautam was celebrating his birthday alongwith some of his friends at home. All of sudden the ceiling fan of the room stopped working. Out of shear enthusiasm, Gautam first switched off the power supply of the fan then opened the cap of the fan to look for the problem. His friend Taushar tried to stop him but he did not pay any attention. As soon as he touched upon the interior parts of the fan, he received a severe electric shock and fell down. All his friends were scared as to what had happened because the power supply had been already switched off.
(*a*) What negative trait has been shown by Gautam ?
(*b*) What could be the possible cause of electric shock ?
(*c*) Write expressions for current and emf of the component used in fan with proper phase difference.

2. One day Priyanka went to the market with her mother in a metro rail. At the metro station, they were made to walk through a doorway of a metal detector for security reasons. Priyanka passed through it and started waiting for her mother to come. She heard a long beep when her mother passed through metal detector. Priyanka was surprised why the metal detector beeped in case of her mother. She asked the duty staff, who told her that it was due to the bunch of metal keys lying in the purse of her mother. Both Priyanka and her mother were satisfied with the security system.
(*a*) What values were displayed by Priyanka ?
(*b*) What is the cause of sound through the metal detector ?
(*c*) On what principle does a metal detector work ?

3. Mohit spent few years in USA and then returned back to India. Once he discussed with his friend Sumit on domestic supply in USA and in India. In USA, domestic power supply is at 110 V, 50 Hz, while in India it is 220 V, 50 Hz. Mohit insisted that USA supply is better than Indian supply. Both went

to Sumit's father who was an electrical engineer and sought his opinion on the issue. He explained that both types of supplies have advantages and disadvantages.

(a) What are the values shown by Mohit and Sumit ?

(b) Give one advantage and one disadvantage of 220 V supply over 110 V supply.

4. Sushil is in the habit of charging his mobile and then leaving the charger connected through the mains with the switch on. When his sister Asha pointed it out to him, he replied there was no harm as the mobile had been disconnected. Asha then explained to him and convinced him, how the energy was still being wasted as the charger was continuously consuming energy.

Answer the following questions :

(a) What values did Asha display in convincing her brother ?

(b) What measures, in your view, should be adopted to minimize the wastage of electric energy in your households ?

(c) Imagine an electric appliance of 2 W, left connected to the mains for 20 hours. Estimate the amount of electrical energy wasted. [CBSE F 15]

5. Ajit had a high tension tower erected on his farm land. He kept complaining to the authorities to remove it as it was occupying a large portion of his land. His uncle, who was a teacher, explained to him the need for erecting these towers for efficient transmission of power. As Ajit realized its significance, he stopped complaining.

Answer the following questions :

(a) Why is it necessary to transport power at high voltage ?

(b) A low power factor implies large power loss. Explain.

(c) Write two values each displayed by Ajit and his uncle. [CBSE OD 15]

6. A group of students while coming from the school noticed a box marked "Danger H.T. 2200 V" at a substation in the main street. They did not understand the utility of such a high voltage, while they argued, the supply was only 220 V. They asked their teacher this question the next day. The teacher thought it to be an important question and therefore explained to the whole class.

Answer the following questions :

(i) What device is used to bring the high voltage down to low voltage of a.c. current and what is the principle of its working ?

(ii) Is it possible to use this device for bringing down the high dc voltage to the low voltage ? Explain.

(iii) Write the values displayed by the students and the teacher. [CBSE D 15]

7. One morning an old man walked bare-foot to replace the fuse wire in kit kat fitted with the power supply mains for his house. Suddenly he screamed and collapsed on the floor. His wife cried loudly for help. His neighbour's son Anil heard the cries and rushed to the place with shoes on. He took a wooden baton and used it to switch off the main supply.

Answer the following questions :

(i) What is the voltage and frequency of mains supply in India ?

(ii) These days most of the electrical devices we use require ac voltage. Why ?

(iii) Can a transformer be used to step up dc voltage ?

(iv) Write two qualities displayed by Anil by his action. [CBSE OD 15]

Answers

1. (a) Careless attitude towards life.

(b) When the circuit breaks, the high induced voltage charges the capacitor. This gives electric shock upon touching.

(c) $\mathcal{E} = \mathcal{E}_0 \sin \omega t$ and $I = I_0 \sin\left(\omega t + \frac{\pi}{2}\right)$.

2. (a) Keen observer, curiosity and quest for knowledge.

(b) When we walk through the doorway of a metal detector carrying some metallic object, the impedance of the circuit changes changing the current significantly. This change in current is detected and sounds as an alarm.

(c) A metal detector works on the principle of resonance in ac circuits.

3. (a) General awareness, critical thinking and curiosity.

(b) *Advantage.* The power loss at 220 V supply is less than that at 110 V.

Disadvantage. The 220 V is more dangerous because its peak value (311 V) is much higher than the peak value (155.5) for 110 V supply.

4. (a) Asha is knowledgeable, convincing, thoughtful and has concern for conservation of resources.

(b) (i) All electrical devices should be switched off when not in use.

(ii) High power devices should be used only when needed.

(c) Electrical energy wasted

$$= P \times t = 2\text{ W} \times 20\text{ h}$$

$$= \frac{2}{1000}\text{ kW} \times 20\text{ h} = \mathbf{0.04\ kWh}\,.$$

or $$E = 2 \times 20 \times 3600\text{ J} = \mathbf{144000\ J}\,.$$

5. (a) For the same power at high voltage, the current in the transmission wires will be small. Hence the power loss ($P = I^2 R$) will be less during transmission.

(b) As $P_{av} = V_{eff}\, I_{eff} \cos\phi$

$$\therefore \quad I_{eff} = \frac{P_{av}}{V_{eff} \cos\phi}$$

To supply a given power, low power factor ($\cos\phi$) requires a larger current to be supplied. This results in large power loss.

(c) Ajit displays social awareness, understanding nature, concern for society.

Uncle is knowledgeable and has professional honesty and concern for society.

6. (i) *Transformer*. It works on the principle of mutual induction *i.e.*, when an alternating current is passed through one of the two inductively coupled coils, an induced emf is set up in the other coil.

(ii) No, there is no induced emf for the dc voltage in the primary.

(iii) *Students*. Inquisitive nature/Scientific temperament.

Teacher. Professional honesty/Concern for students/Helpfulness.

7. (i) Voltage = 220 V and frequency = 50 Hz.

(ii) (a) AC voltage can be easily stepped up/stepped down.

(b) It can be easily converted into dc.

(c) Line losses can be minimised by using ac voltage.

(iii) No, there is no induced emf for the dc voltage in the primary.

(iv) Anil is helping, brave and has knowledge of insulators, conductors and safety precautions.

COMPETITION SECTION

Alternating Current and Electrical Machines

GLIMPSES

1. **Alternating current.** It is that current which varies in magnitude continuously and reverses its direction periodically. Its value at any instant is given by

$$I = I_0 \sin \omega t = I_0 \sin 2\pi ft$$

where I_0 is the peak value of a.c. or current amplitude. The frequency of a.c. supplied to our homes is 50 cps. The average value of a.c. over a complete cycle is zero.

2. **Direct current.** It is that current which flows with a constant magnitude in the same fixed direction.

3. **Average or mean value of a.c.** It is that value of direct current which sends the same charge in a circuit in the same time as is sent by the given alternating current in the same circuit in its half time period.

$$I_{av} = \frac{2}{\pi} I_0 = 0.637\ I_0$$

4. **Effective or rms or virtual value of a.c.** It is that value of direct current which produces the same heating effect in a given resistor as is produced by the given alternating current when passed for the same time.

$$I_{rms} \quad \text{or} \quad I_{eff} \quad \text{or} \quad I_v = \frac{1}{\sqrt{2}} . I_0 = 0.707\ I_0$$

5. **Alternating voltage.** It is that voltage whose magnitude varies continuously and direction reverses periodically with time. Its instantaneous, average and root mean square values are respectively given by

$$\mathcal{E} = \mathcal{E}_0 \sin \omega t, \quad \mathcal{E}_{av} = 0.637\ \mathcal{E}_0,$$

$$\mathcal{E}_{rms} = \frac{1}{\sqrt{2}} \mathcal{E}_0$$

6. **Phasors and phasor diagrams.** A rotating vector that represents a sinusoidally varying quantity is called a phasor. A diagram that represents alternating current and voltage of the same frequency as rotating vectors (phasors) alongwith proper phase angle between them is called a phasor diagram or Argand diagram.

7. **A.C. circuit containing resistor only.** An alternating voltage, $\mathcal{E} = \mathcal{E}_0 \sin \omega t$ applied to a resistor R drives a current $I = I_0 \sin \omega t$ in the resistor. The current is in phase with the applied voltage.

Peak value of current, $I_0 = \dfrac{\mathcal{E}_0}{R}$

8. **A.C. circuit containing only an inductor.** An alternating voltage, $\mathcal{E} = \mathcal{E}_0 \sin \omega t$ applied to a pure inductor L drives a current, $I = I_0 \sin(\omega t - \pi/2)$ in the inductor. The current in the inductor lags behind the voltage in phase by $\pi/2$ rad.

Peak value of current, $I_0 = \dfrac{\mathcal{E}_0}{\omega L} = \dfrac{\mathcal{E}_0}{X_L}$

Root mean square value of current,

$$I_{rms} = \frac{\mathcal{E}_{rms}}{X_L} = \frac{\mathcal{E}_{rms}}{\omega L} = \frac{\mathcal{E}_0}{\sqrt{2} . \omega L}$$

9. **Reactance.** The non-resistive opposition to the flow of a.c. is called reactance. It may be inductive reactance (X_L) or capacitive reactance (X_C).

10. **Inductive reactance.** It is the effective resistance or opposition offered by an inductor to the flow of a.c. through it. It is given by

$$X_L = \omega L = 2\pi fL$$

The SI unit of inductive reactance is ohm (Ω).

For a.c., $X_L \propto f$

For d.c., $f = 0$, so $X_L = 0$.

11. A.C. circuit containing only a capacitor. An alternating voltage, $\mathcal{E} = \mathcal{E}_0 \sin \omega t$ applied to a capacitor C drives a current, $I = I_0 \sin(\omega t + \pi/2)$ in the capacitor. The current in the inductor is ahead of voltage in phase by $\pi/2$ rad.

Peak value of current,

$$I_0 = \frac{\mathcal{E}_0}{1/\omega C} = \frac{\mathcal{E}_0}{X_C}$$

Root mean square value of current,

$$I_{rms} = \frac{\mathcal{E}_{rms}}{X_C} = \frac{\mathcal{E}_{rms}}{1/\omega C} = \frac{\mathcal{E}_0}{\sqrt{2}\,.\,1/\omega C}$$

12. Capacitive reactance. It is the effective resistance or opposition offered by a capacitor to the flow of a.c. through it.

It is given by

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$$

The SI unit of capacitive reactance is ohm (Ω)

For a.c., $X_C \propto \frac{1}{f}$

For d.c., $f = 0$, so $X_C = \infty$

Thus a capacitor allows a.c. to flow through it easily but blocks d.c.

13. Impedance. It is a quantity that measures the opposition of a circuit to the flow of current through it and so determines the magnitude of the current. In a d.c. circuit, this is the resistance R alone. In an a.c. circuit, the reactance X also has to be taken into account, according to the relation :

$$Z^2 = R^2 + X^2$$

where Z is the impedance. ***Impedance triangle*** is a right angled triangle whose base represents resistance R, perpendicular represents reactance X and hypotenuse represents impedance Z of the circuit. From this triangle, the phase angle ϕ between voltage and current is given by

$$\tan \phi = \frac{X}{R} \quad \text{or} \quad \cos \phi = \frac{R}{Z}.$$

14. A.C. through a series *LR*-circuit. The alternating voltage leads the current by a phase angle ϕ.

Instantaneous voltage, $\mathcal{E} = \mathcal{E}_0 \sin \omega t$

Instantaneous current, $I = I_0 \sin(\omega t - \phi)$

Peak current,

$$I_0 = \frac{\mathcal{E}_0}{\sqrt{R^2 + X_L^2}} = \frac{\mathcal{E}_0}{Z}$$

Impedance,

$$Z = \sqrt{R^2 + X_L^2} = \sqrt{R^2 + \omega^2 L^2}$$

Phase angle,

$$\phi = \tan^{-1} \frac{X_L}{R} \quad \text{or} \quad \phi = \cos^{-1} \frac{R}{Z}.$$

15. A.C. through a series CR-circuit. The alternating voltage lags behind the voltage by phase angle ϕ.

Instantaneous voltage, $\mathcal{E} = \mathcal{E}_0 \sin \omega t$

Instantaneous current, $I = I_0 \sin(\omega t + \phi)$

Peak current, $I_0 = \frac{\mathcal{E}_0}{\sqrt{R^2 + X_C^2}} = \frac{\mathcal{E}_0}{Z}$

Impedance, $Z = \sqrt{R^2 + X_C^2} = \sqrt{R^2 + \omega^2 L^2}$

Phase angle, $\phi = \tan^{-1} \frac{X_C}{R}$ or $\phi = \cos^{-1} \frac{R}{Z}$.

16. The series *LCR*-circuit. For a series *LCR*-circuit connected across a source $\mathcal{E} = \mathcal{E}_0 \sin \omega t$, the current I is given by

$$I = I_0 \sin(\omega t - \phi) = \frac{\mathcal{E}_0}{Z} \sin(\omega t - \phi)$$

where Z is the total effective resistance of the *LCR*-circuit and is called its impedance.

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

The phase angle ϕ between voltage and current is given by

$$\tan \phi = \frac{X_L - X_C}{R} \quad \text{and} \quad \cos \phi = \frac{R}{Z}$$

The voltage leads the current if $X_L > X_C$ and it lags behind the current if $X_L < X_C$.

17. Resonance condition of the *LCR*-circuit. If $X_L = X_C$, the impedance of *LCR* circuit becomes

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = R$$

The impedance is minimum and hence current is maximum. The circuit is purely resistive and voltage and current are in same phase. This is the resonance condition of *LCR*-circuit and is satisfied at the resonant frequency given by

$$f_r = \frac{1}{2\pi\sqrt{LC}}.$$

18. Q-Factor. It indicates the sharpness of resonance. It is defined as the ratio of the resonant frequency to the difference in two frequencies taken on both sides of the resonant frequency such that at each frequency, the current amplitude becomes $1/\sqrt{2}$ times the value at resonant frequency.

$$Q\text{-factor} = \frac{\text{Resonant frequency}}{\text{Band width}}$$

$$= \frac{\omega_r}{\omega_2 - \omega_1} = \frac{\omega_r L}{R} = \frac{1}{R}\sqrt{\frac{L}{C}}.$$

19. Power in an a.c.-circuit. The average power in an a.c. circuit consumed per cycle is given by

$$P_{av} = \mathcal{E}_{rms}\, I_{rms} \cdot \cos\phi = \frac{\mathcal{E}_0 I_0}{2}\cos\phi$$

Here $\cos\phi$ is called ***power factor*** which is the ratio of true power (P_{av}) to the apparent power ($\mathcal{E}_{rms}\, I_{rms}$).

Power factor, $\cos\phi = \dfrac{R}{Z} = \dfrac{R}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}$

(*i*) For a *pure resistive circuit*, $\phi = 0$,

$$P_{av} = \mathcal{E}_{rms}\, I_{rms} = \frac{\mathcal{E}_{rms}^2}{R}.$$

(*ii*) For a pure inductive circuit, $\phi = \dfrac{\pi}{2}$, $P_{av} = 0$.

(*iii*) For a pure capacitive circuit, $\phi = -\dfrac{\pi}{2}$, $P_{av} = 0$.

20. Wattless current. The current in an a.c. circuit is said to be wattless if the average power consumed in the circuit is zero. It is the component $I_{rms}\sin\phi$ of the alternating current. In an inductive or capacitive a.c. circuit, the phase difference between voltage and current, $\phi = \pm\pi/2$. Power factor $\cos\phi = 0$ and so the current is wattless.

21. Energy stored in an inductor. When the current in an inductor grows from 0 to I_0, the magnetic energy stored in it is

$$U = \frac{1}{2} L I_0^2$$

22. Energy stored in a capacitor. The energy stored in a capacitor when it is charged from 0 to V volt is

$$U = \frac{1}{2} CV^2 = \frac{1}{2} QV = \frac{1}{2}\frac{Q^2}{C}.$$

23. *LC*-oscillations. When a charged capacitor is allowed to discharge through a non-resistive inductor, electrical oscillations of constant amplitude and frequency are produced. These oscillations are called *LC*-oscillations. The charge of the capacitor satisfies the equation of SHM :

$$\frac{d^2q}{dt^2} + \frac{1}{LC} q = 0$$

Instantaneous charge, $q = q_0 \cos\omega_0 t$

Instantaneous current, $I = -\dfrac{dq}{dt} = \omega_0 q_0 \sin\omega_0 t$

Angular frequency of free oscillations,

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

Frequency of free oscillations,

$$f_0 = \frac{\omega_0}{2\pi} = \frac{1}{2\pi\sqrt{LC}}$$

The energy in the *LC*-circuit oscillates between the capacitor and the inductor but the total energy remains constant.

24. Choke coil. It is an inductor with large inductance used to reduce current in an a.c. circuit without much loss of power.

25. Transformer. It is a device used to convert a.c. at high voltage into that at low voltage or vice versa. For an ideal transformer,

$$\frac{\mathcal{E}_2}{\mathcal{E}_1} = \frac{I_1}{I_2} = \frac{N_2}{N_1} = k$$

Here suffix 1 refers to primary coil and suffix 2 to secondary coil and k is called *transformation* or *turns ratios* of the transformer.

(*i*) *For a step up transformer*, $k > 1$ or $N_2 > N_1$

$\therefore \quad \mathcal{E}_2 > \mathcal{E}_1$ and $I_2 < I_1$.

(*ii*) *For a step down transformer*, $k < 1$ or $N_2 < N_1$

$\therefore \quad \mathcal{E}_2 < \mathcal{E}_1$ and $I_2 > I_1$.

26. A.C. generator. It is a device to convert mechanical energy into electrical energy of alternating form. It consists of a coil of wire that rotates with angular velocity ω in the magnetic field B of a permanent magnet. The flux through the coil varies as $\phi = NBA\cos\omega t$, where N is the number of turns in the coil having the face area A. (At $t = 0$, the loop is normal to the field).

Induced emf, $\mathcal{E} = NBA\,\omega\sin\omega t = \mathcal{E}_0 \sin\omega t$

Current, $I = \dfrac{NBA\,\omega}{R}\sin\omega t = I_0 \sin\omega t.$